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Journal

of

THE ROYAL AERONAUTICAL SOCIETY

OCTOBER 1953

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Journal of

THE ROYAL AERONAUTICAL SOCIETY

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

PUBLISHED MONTHLY AT

4 HAMILTON PLACE, LONDON, W.1

Telephone: Grosvenor 3515-9 *Telegrams*: Didaskalos, Audley, London

SUBSCRIPTIONS: £6 6s. 0d. PER ANNUM, POST FREE. SINGLE COPIES: 10s. 6d., POST FREE.

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All communications for publication in, and advertisements for, the Journal should be addressed to the Editor, Joan Bradbrooke, A.R.Ae.S.; all communications on general matters affecting the Society should be addressed to the Secretary, A. M. Ballantyne, T.D., B.Sc., Ph.D., A.F.R.Ae.S.

None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless stated otherwise.

PRINTED BY THE LEWES PRESS (WIGHTMAN & CO. LTD.), LEWES, SUSSEX, ENGLAND, AND PUBLISHED BY THE ROYAL AERONAUTICAL SOCIETY, 4 HAMILTON PLACE, LONDON, W.1, ENGLAND.

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The Journal of the Royal Aeronautical Society

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

VOLUME 57 OCTOBER 1953 NUMBER 514

LUDWIG PRANDTL

WITH THE DEATH OF LUDWIG PRANDTL, *Honorary Fellow*, which occurred at Göttingen on 15th August 1953, the aeronautical world has lost one who was perhaps the most eminent of its scientific pioneers. It would scarcely be going too far to call him the Grand Old Man of aerodynamic theory, for there is hardly a branch of it to which he did not contribute, and many of its fundamental concepts originated in his fertile mind.

Born in 1875, he was educated as an engineer and occupied several posts in the teaching profession before he went to Göttingen in 1907 and there founded the school of hydrodynamics and aerodynamics that has since become famous thoughout the world. He directed this school until age compelled him to retire, and by his researches and his inspired teaching he did at least as much as any man to lay the foundations of the science of aeronautics.

In addition to his great mathematical powers, which enabled him to open up new paths in his theoretical work, Prandtl was a very able experimenter and could devise and carry out crucial experiments to put his theories to the test. He was, indeed, the first to design a wind tunnel with a very steady air flow by the use of the now well-known devices of a fine honeycomb and a rapid contraction before the working section. In this he tested some of the first airship models and explored the critical behaviour of the drag of spheres, in which latter work he seems to have been the first to appreciate the effect of stream turbulence on the boundary layer round a body. His theory of the wing of finite span, and the formulation of the induced drag, is classic and is the basis of much modern work on lift distribution and its effects on drag.

Probably his greatest contribution to theory was his work on the boundary layer, and the reduction of the general equations of viscous flow to a form applicable in the thin boundary layer and capable of analytical solution. On this foundation has grown an enormous amount of research which enables us today to calculate skin friction in a great variety of cases, provided we know where transition occurs. He did not solve the problem of predicting where transition from laminar to turbulent flow would occur under given circumstances, nor have we solved it yet. His work on the flow of compressible fluids was again the starting point of innumerable studies by other workers and has led to great advances in our knowledge of transonic and supersonic aerodynamics. In addition to this vast amount of individual and fundamental research, he was a great teacher and his pupils at Göttingen include some of the most illustrious names in present-day aeronautics. His writings, too, inspired other writers in many countries, though it was often a long time before the significance of his discoveries was fully appreciated in other parts of the world.

Courtesy of Zeitschrift für Flugwissenschaften

He was a modest man but with a personality of great charm which made him a pleasant companion to those who were fortunate enough to come to know him. During the last years of his life he suffered greatly in health; but for this we might perhaps have been given the clue to some of the problems which still baffle us, for it was Prandtl's way to see the essence of things and to start a train of thought that led to great advances by others who followed it.

The value of Prandtl's work is now so well known that it is almost superfluous to say that he has probably done more than any other single man to advance the development of aeronautical engineering by providing a sound fundamental basis on which to build. He received many honours and recognitions of his work both in his own country and abroad, and the esteem in which he was held in this country is shown by the fact that he received the Gold Medal of the Royal Aeronautical Society, Honorary Fellowship of the Royal Aeronautical Society, the Foreign Membership of the Royal Society, and an honorary degree of the University of Cambridge. Many of us will mourn the loss of a personal friend, while still more will realise that from our midst has passed a pioneer who was without equal in the field of aeronautical theory and experiment.—E. F. RELF.

NOTICES

Royal Honorary Fellowship

The Council of the Royal Aeronautical Society have pleasure in announcing that His Royal Highness the Duke of Edinburgh, K.G., has graciously accepted *Honorary Fellowship* of the Society.

This Honour has also been conferred on the following:

Sir Geoffrey de Havilland, C.B.E., Hon.F.I.A.S.

Sir Arthur Gouge, B.Sc., M.I.Mech.E.

Lord Hives, C.H., M.B.E., B.Sc.

Honours Awarded to Members

The Council of the City and Guilds of London Institute have conferred the honour of Fellowship on Mr. L. P. Coombes, *Fellow* of the Society.

Associate Fellowship Examination

The next Associate Fellowship Examination will be held in the offices of the Society on 8th, 9th and 10th December 1953. All candidates who have entered for the Examination will be informed individually of the detailed arrangements.

Members' New Appointments

G. R. Edwards (*Fellow*) has been appointed Managing Director of the Aircraft Division of Vickers-Armstrongs Ltd.

Rear-Admiral Caspar John (*Fellow*) has been appointed Deputy Controller of Supplies (Air). He was formerly Chief Naval Representative.

Dr. W. P. Jones (*Associate Fellow*) has been appointed Superintendent of the Aerodynamics Division of the National Physical Laboratory.

Harald J. Penrose (*Fellow*), for some years the Chief Test Pilot of Westland Aircraft, has now been appointed Sales Manager of that firm.

J. C. K. Shipp (*Associate Fellow*) has recently been appointed the English Electric Company's representative in the United States. He was formerly with the Ministry of Supply.

W. W. Warner (*Associate Fellow*) has been appointed Factory Manager of the Airspeed Division of the de Havilland Aircraft Company. He was formerly Service Manager of that division.

Commonwealth Fund Fellowships—for Advanced Study and Travel in the United States

Twenty Fellowships are offered in 1954 to men and women who are British subjects, not normally resident in or near the Americas, and who are degree graduates of a university in the United Kingdom of Great Britain and Northern Ireland. Candidates must be available for interview in London in March 1954.

Conditions of eligibility, award and tenure and the emoluments offered for each category of the Fellowships are published in separate memoranda. Applications should be submitted on the prescribed form. The appropriate memorandum with application form attached may be obtained through British universities or through Government departments, or from The Warden, Harkness House, 35 Portman Square, London, W.1. Applications must be returned by 15th December 1953.

Performance Data Sheets—A Correction

A misprint has been pointed out in Data Sheet EG3/1 "Estimation of Rate of Climb." On page one, equation (2), the coefficient of $KW^2/(b^2V_1)$ should read 0·2882 and not 0·882 as printed.

Society of British Aircraft Constructors' Educational Grants

The following were awarded Educational Grants by the Society of British Aircraft Constructors at a Selection Committee meeting held in the offices of the Royal Aeronautical Society on 29th July 1953:—

D. M. Abraham, 130 Lincoln Road, Newark, Notts. (To serve his apprenticeship with Blackburn and General Aircraft.)

I. R. Bayley, 65 Panfield Lane, Braintree, Essex. (To serve his apprenticeship with the de Havilland Engine Co.)

D. M. Bloor, 35 Hawkesbury Road, Canvey Island, Essex. (To serve his apprenticeship with the Fairey Aviation Co.)

M. W. Davidson, 36 Worlds End Avenue, Quinton, Birmingham 32. (Awarded the Sir Nigel Norman Scholarship.) (To serve his apprenticeship with the Fairey Aviation Co.)

D. L. Marsden, 22 St. Columba Road, Bridlington, E. Yorks. (To serve his apprenticeship with Blackburn and General Aircraft.)

M. H. Radford, 16 St. Margaret's Crescent, Leiston, Suffolk. (Awarded the John de Havilland Scholarship.) (To serve his apprenticeship with the de Havilland Aircraft Co. Ltd.)

C. M. W. H. Roper, 58 Kidmore Road, Caversham, Reading. (To serve his apprenticeship with Saunders-Roe Ltd.)

F. A. Smith, 2 Bentley Avenue, Lightcliffe, Halifax, Yorks. (To serve his apprenticeship with the de Havilland Aircraft Co. Ltd.)

The Scholarship Selection Board consisted of representatives of the Royal Aeronautical Society and the Society of British Aircraft Constructors.

Graduates' and Students' Section
Annual Reception and Dance

The Annual Reception and Dance of the Students' and Graduates' Section will be held at the Society on Friday, 30th October 1953 from 7.30-11 p.m. Tickets, price 6/- each, including refreshments are obtainable from Mr. H. S. Mettam, 5 Lynmouth Gardens, Heston, Middlesex. Members of the Section are requested to enclose a stamped and addressed envelope with their remittance.

Graduates' and Students' Section

Because of the heavy demand from members wishing to visit the Royal Aircraft Establishment in August a similar visit has been arranged for Wednesday, 2nd December 1953, from 10.30 a.m. to 4.30 p.m. Lunch will be available. Visitors will be able to see the wind tunnels, structural and engine test rigs, and the flight test department.

Applications should be sent to the Hon. Visits Secretary, D. J. W. Richards, 18 Geneva Road, Kingston-on-Thames, Surrey. Members are asked to state their grade of membership of the Society and that they are British Subjects. Applications must be sent as soon as possible as numbers are strictly limited.

Diary

LONDON

October 13th

SECTION LECTURE. The Use of Materials in the Plastics Range. H. L. Cox, M.A., F.R.Ae.S. In the Library, 4 Hamilton Place, W.1. 7 p.m.

October 20th

GRADUATES' AND STUDENTS' SECTION. An Introduction to Gas Turbines. D. H. Mallison. 4 Hamilton Place, W.1. 7.30 p.m.

October 29th

MAIN LECTURE. A Quantitative Study of Instrument Approach. J. F. W. Mercer. At the Institution of Mechanical Engineers, Storey's Gate, S.W.1. 6 p.m. (Tea at 5.30 p.m.)

October 30th

GRADUATES' AND STUDENTS' SECTION. Informal Reception and Dance. 4 Hamilton Place, W.1. 7.30 to 11 p.m. *By ticket only* See Notice.

November 3rd

SECTION LECTURE. The Development of 4,000 lb. Hydraulic Systems. G. Orloff, A.F.R.Ae.S. In the Library, 4 Hamilton Place, W.1. 7 p.m.

November 5th

GRADUATES' AND STUDENTS' SECTION. The Exploration of Space. Arthur C. Clarke, B.Sc., F.R.A.S. 4 Hamilton Place, W.1. 7.30 p.m.

November 10th

SECTION LECTURE. Gusts and Their Measurement. J. Taylor, M.A., A.F.R.Ae.S. In the Library, 4 Hamilton Place, W.1. 7 p.m.

November 12th

MAIN LECTURE. Stability and Control in Aircraft Design. J. C. Wimpenny, Grad.R.Ae.S. At the Institution of Mechnical Engineers, Storey's Gate, S.W.1. 6 p.m. (Tea at 5.30 p.m.)

November 19th

SECTION LECTURE. Aircraft Stability and Control. A. W. Babister, M.A., A.F.R.Ae.S. In the Library, 4 Hamilton Place, W.1. 7 p.m.

November 26th

GRADUATES' AND STUDENTS' SECTION. The Case for the Light Fighter. P. A. Norman, B.Sc., Student R.Ae.S. 4 Hamilton Place, W.1. 7.30 p.m.

December 1st

SECTION LECTURE. Integral Construction. K. L. C. Legg. In the Library, 4 Hamilton Place, W.1. 7 p.m.

BRANCHES

October 7th

Bristol.—Flutter. Professor W. J. Duncan. Conference Room, Bristol Aeroplane Co. Ltd., Bristol. 6 p.m.

Chester.—Requirements of Powered Flying Controls. H. G. Conway. Grosvenor Hotel, Chester. 7.30 p.m.

Luton.—Aerial Surveying. Captain Saffrey. George Hotel. 7.30 p.m.

October 10th

Halton.—Annual General Meeting at 16.00 hours followed by Flight Testing of Flying Boats. Geoffrey Tyson. Branch Hut, R.A.F. Station, Halton.

October 12th

Henlow.—Presidential Address: Air Logistics. Air Chief Marshal Sir John Whitworth Jones. R.A.F. Technical College, Henlow. 7.30 p.m.

October 17th

Chester.—Visit to de Havilland Aircraft Co. Ltd., Broughton. 2.30 p.m.

October 19th

Henlow.—Parachute Progress and Design. S. B. Jackson. R.A.F Technical College, Henlow. 7.30 p.m.

October 21st

Manchester.—Some Aspects of High Speed Jet Aircraft. Sqdn. Ldr. W. Waterton, G.M., A.F.C. Reynolds Hall, College of Technology, Manchester. 7.30 p.m.

October 23rd

Birmingham.—Helicopters. Birmingham Chamber of Commerce, Birmingham. 7.30 p.m.

October 26th

Halton.—Junior Members' Night. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

November 2nd

Henlow.—Titanium, Some Aircraft Applications. Major P. L. Teed. R.A.F. Technical College Henlow. 7.30 p.m.

Halton.—Films. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

November 3rd

Bristol.—Film Show. Conference Room, Bristol Aeroplane Co. Ltd. 6 p.m.

November 4th

Chester.—The Proteus. Dr. S. G. Hooker. Grosvenor Hotel, Chester. 7.30 p.m.

Luton.—Film and Discussion Evening. George Hotel. 7.30 p.m.

November 9th

Halton.—Petroleum and the Aircraft Industry. K. C. Hunt. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

November 13th

Chester.—Annual Dance at Quaintways Restaurant, Chester.

November 16th

Halton.—Branch Night. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

November 18th

Manchester.—Recent Development in Gliding. A. H. Yates. Reynolds Hall, College of Technology, Manchester. 7.30 p.m.

November 23rd

Halton.—Junior Members' Night. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

November 24th

Henlow.—Films. R.A.F. Technical College, Henlow. 7.30 p.m.

November 25th

Bristol.—Operational Requirements in the R.A.F. Air Cdre. W. H. Kyle. Conference Room, Bristol Aeroplane Co. Ltd., Filton. 6 p.m.

November 27th

Birmingham.—Some Uses for the Small Gas Turbine. Birmingham Chamber of Commerce. 7.30 p.m.

November 30th

Halton.—Films. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

Scale Model of BE 2C Presented by Colonel O'Gorman

The Council gratefully acknowledge the gift of a silver model BE 2C presented by Colonel Mervyn O'Gorman. The BE 2C (1913) was the first inherently stable aeroplane. Stability calculations and tests were made by E. T. Busk, who was awarded the Gold Medal of the Aeronautical Society in 1915. About 2,000 BE2C's were used in the 1914-18 War and were made to the same working drawings as this model.

Acknowledgments

The Council acknowledge with thanks the return of back numbers of the JOURNAL from Miss A. D. Betts, *Companion;* S. R. Brown, Esq., *Associate Fellow;* The Ministry of Supply, at the Royal Aircraft Establishment; and C. Gilford, Esq., *ex Associate Fellow.*

Annual Subscriptions

Members are reminded that their annual subscriptions become due on 1st January 1954. The rates are:—

	Home	Abroad
	£ s. d.	£ s. d.
Fellows	5 5 0	4 4 0
Associate Fellows	4 4 0	3 3 0
*Associates	3 3 0	3 3 0
Graduates (aged under 26)	2 2 0	2 2 0
Graduates (aged 26 and over)	2 12 6	2 12 6
Students (aged under 21)	1 1 0	1 1 0
Students (aged 21 and over)	1 11 6	1 11 6
Companions	3 3 0	3 3 0
Founder Members	2 2 0	2 2 0

* Any Associate elected before 1st October 1947 may, if he wishes, elect not to receive the JOURNAL, and in this case his subscription will be reduced by £1 1s. 0d. to £2 2s. 0d.

It will avoid delay and confusion if members, when sending remittances for subscriptions, will state their names clearly and give their addresses and grades of membership. Remittances should be made payable to the Royal Aeronautical Society.

Changes of Address

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address.

When notifying changes please give the following particulars:—

Name (in block letters).

Grade of membership.

New address (in block letters).

Old address.

Changes of address must be received *before the 15th of the month in order to be effective for the* JOURNAL for the following month.

Elections

The following is a list of new members and transfers of membership of the Society:—

Associate Fellows

Frederick Henry Carey (*ex Student*)
William Harry Casley (*from Graduate*)
Edward Allen Crabbe
Larratt Cassels Darling
Ulric Keith Gerry (*from Graduate*)
Leslie James Green (*from Graduate*)
Gordon Sinclair Henderson (*from Graduate*)
Mostyn David Kimber (*from Graduate*)
Vaidy Samarapuri Ponnaranga Mudaliar
Herbert Dorman Spreckley
Stanley Cecil Thorp (*from Associate*)
Lloyd Brian Walker.

Associates

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The Journal of the Royal Aeronautical Society

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

VOLUME 57 OCTOBER 1953 NUMBER 514

Estimation of the Fatigue Life of a Transport Aircraft

by

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SUMMARY:—A simple formula is presented for the safe life of an aircraft under reasonably good operating conditions from the standpoint of wing fatigue. Gusts are treated as the main factor in determination of fatigue life, though other factors are allowed for as being secondary. Before the formula can be used a simple fatigue test is required on critical components, and is repeated several times to establish variation of nominally identical specimens. The loading cycle for this test corresponds to a sequence of up and down gusts of 8 ft./sec. equivalent velocity. The life obtained is termed the "Standard Life" and corresponds to normal operating conditions on European air lines. Reservations in the use of the formula and correction for other operating conditions are discussed in general terms.

1. Introduction

In the past few years general structural fatigue has become a major design consideration for transport aircraft. For a much longer period, of course, fatigue has been a possible danger, but in the earlier times it was associated with some mechanical or aerodynamic imperfection. Unfortunate engine or propeller resonances, for example, have always had to be watched closely from the fatigue standpoint, as also have vibrations caused by buffeting and incipient flutter.

The significance of the present phase of fatigue history is that fatigue can occur as the result of the ordinary wear-and-tear of an aircraft's existence. Gusts are generally the main contributor to this kind of fatigue, but they need only be gusts of a kind normally accepted as in no way exceptional. It is their cumulative effect which is significant.

This state of affairs was envisaged as a future possibility in a paper presented by the author some five years ago[1]. At that time the situation could not be regarded as serious, but since then it has tended to become critical. In the explanation for this change, all-weather flying and the high utilisation rate achieved on modern air lines are vital factors.

A simple empirical formula for the safe life of an aircraft is the objective in the present paper. If the safe life can be predicted for the appropriate operating conditions then fatigue becomes essentially an economic issue[2]. The aim is primarily to enable fatigue life to be predicted early, at a stage when the estimate need only be approximate, though naturally erring on the side of safety. More exact calculations can be made later on the basis of *ad hoc* research in the laboratory and in the air. At present, fatigue of the wing structure normally determines the fatigue life, and consideration is confined to wings in this paper.

It would be desirable to predict fatigue life directly from calculations supported by routine data. Unfortunately this is not yet possible, and some *ad hoc* testing of the actual aircraft type is necessary. This does not lessen the task of deriving a simple formula for fatigue life, however, since the need to keep testing within reasonable limits has to be a major consideration. The method here put forward calls for a single test on all critical components, repeated only as necessary to establish variation of nominally identical specimens. It would be easy to produce more accurate formulae at the price of a testing programme exceeding the practical limitations on cost, research effort, and time.

The formula given has been obtained by the trial-and-error process usual in such cases, in which direct reasoning and scientific deduction are amalgamated with practical experience. The process was described on general historical lines in an earlier paper by the author[3]. In the present paper, which is in some respects a sequel to the previous one, a more direct approach is taken: the main assumptions are first laid down, and from these the formula for life is developed directly, with empirical factors introduced at appropriate stages. Use is made of recent experience and operational data not previously available.

The estimate of safe life obtained by this method is referred to as the "Standard Life." This corresponds to what may be regarded from the fatigue standpoint

Received 11th August 1953.

as reasonably good operating conditions, actually those normally obtaining on European air lines. For general use the estimate has to be used with discretion, and corrections have to be introduced for conditions differing widely from those envisaged. The reservations and corrections are discussed generally at the end of this paper.

The "Standard Life" is thus seen to be in certain respects merely a design criterion that could be expressed in other forms. It is clearly convenient, however, to have a criterion which gives directly, even though only roughly, an indication of the life to be expected. The only danger is that the estimate might be accepted as a safe life without regard to the particular operating conditions.

The formula for life permits direct comparison with results of the more accurate methods developed by Taylor[1] for use later in the life of an aircraft when accelerometer data are available. An aircraft of good design should have many years to go before fatigue becomes critical, and this leaves time for more exact analysis. It is not unlikely, moreover, that in time every individual aircraft will be fitted with a "Fatigue Meter" for recording its entire loading history. The empirical formula is expressed in such a form that it does not need to be re-cast when such a record is available.

2. The Main Assumptions

There are four main assumptions on which the formula for "Standard Life" is based. The first is that gusts are the main fatigue hazard. For this assumption there is now ample support in all but exceptional cases, from general experience and flight measurements.

The postulation of this assumption does not mean that other effects are completely ignored, but that gusts dictate the form of the formula. Allowance is made for the other effects in the selection of suitable empirical constants. Furthermore, in the tests on which the life estimate is based, an over-riding minimum loading is imposed for cases where gust loads are abnormally low.

The second assumption is that practically all the fatigue damage caused by gusts is produced by gusts within a fairly narrow velocity range, say from 5 to 15 equivalent feet per second. For this assumption there is now strong evidence, provided that the structure meets the normal gust requirements (50 ft./sec. gust) for static strength. The main significance of this assumption is its bearing on the third assumption next to be considered.

The third assumption is that the fatigue effect of the complex fluctuations of loading occurring in actual flight can be represented by the effects of a single loading cycle applied repeatedly to destruction. Taken alone this assumption implies that endurance curves (alternating loading against cycles to destruction) have much the same general shape for all aircraft structures; so that a single point virtually defines a complete endurance curve. It also implies that more is known than is actually so about the cumulative law for mixed cycles, and about the relationship between random load fluctuations and repeated loading cycles.

The scope of these implications is considerably reduced by the previous assumption that the gusts doing most damage lie within a narrow range. In the test on which the life assessment is based, the alternating load corresponds to gusts roughly in the middle of this range. Nevertheless, this third assumption is the most difficult to substantiate by direct evidence, and it is necessary to bear in mind that the formula for life is approximate and empirical. Ultimate justification of the assumption must rest in the validity of the formula as established by experience. In this respect consistency rather than absolute values is the main consideration, since absolute values depend upon an empirical factor which has also to take into consideration the severity of operating conditions. Experience of the use of the formula is still being acquired, but so far the indications are that it is reasonably reliable. So far as this third assumption is concerned, moreover, variations in fatigue properties and in operating conditions appear to be more serious sources of error.

The fourth and final assumption is the inevitable one that the gust conditions foreseen for the life assessment are known. There is little that can be said about this assumption, since the effect of gust magnitudes and frequencies have to be covered ultimately by an overall empirical factor adjusted in the light of experience. It is of interest to note that for the purpose of life assessment the complex gust conditions of reality are reduced to a single pair of equal up and down gusts occurring a given number of times in a given distance. This procedure, however, is a natural corollary to the third assumption requiring representation of a complex gust loading spectrum by a series of repeated cycles.

3. The Fatigue Test

The fatigue test on which life assessment is based has a prescribed alternating loading superimposed on the loads occurring in level flight. In principle, the magnitude of the alternating loading is that actually imposed on the aircraft in flight when it encounters a gust of 8 ft./sec. equivalent ("indicated") velocity. For this calculation the aircraft is taken to be flying at its normal operating equivalent speed. The value of 8 ft./sec. for the gust velocity is chosen as lying roughly in the middle of the range of greatest fatigue damage. This gust loading is applied first in one direction and then in the other to give the complete alternating cycle, and this cycle is then applied until fatigue failure occurs.

To this loading case it is necessary to apply a factor of safety. Owing to the flatness of the endurance curve (alternating load against cycles to destruction) in the region of the test, it is unsafe to rely solely on a factor of safety applied directly to life. A small change in fatigue strength may produce an entirely disproportionate change in life expressed as a number of cycles.

The factor of safety chosen is 1·25, and is based on general experience. An appreciably lower figure might be unsafe unless compensated by an unduly high overall factor on life. On the other hand, a higher figure might lead to unrepresentative failure since different loading strengths tend to affect different parts in different ways. It is interesting to remark that fatigue life is not so sensitive to variation in mean or steady load. In

consequence no direct factor of safety is required on the steady loading.

In addition to the factor of safety on alternating load a further precaution is necessary. In cases where the gust loading is unusually low, effects that are normally secondary may become unduly significant. Examples are such effects as those produced by landing, take-off, and taxying. An arbitrary over-riding minimum is therefore prescribed, and taken to be 0·25 times the loading produced in undisturbed level flight. This figure is based on experience and is believed to make the formula for life more reliable.

The complete specification for the fatigue test may be summarised as follows.

Mean or Steady Loading

The loading occurring in undisturbed level flight at the operating equivalent speed.

Alternating Loading (Normal)

$\pm 1{\cdot}25$ times the loading superimposed by a gust of 8 ft./sec. equivalent velocity at the operating equivalent speed.

Alternating Loading (Over-riding Minimum)

$\pm 0{\cdot}25$ times the loading produced in undisturbed level flight.

4. Treatment of Test Results—" Nominal Endurance "

A single test under the conditions just described gives a value for the endurance (n) or number of cycles to destruction. Variation in nominally identical specimens, however, is usually considerable and several similar specimens have to be tested. Six is usually regarded as the minimum acceptable.

Now the ultimate objective is to estimate the safe life of an aircraft type. The next step, therefore, is to assess what can be regarded as the safe minimum value (N) for test endurance. This value is taken to be two-thirds of the logarithmic mean (M).

In the choice of this value the flatness of the endurance curve is one of the main considerations. Ideally it would be desirable to test all specimens to alternating loads giving the same endurance, so that variation of strength instead of endurance could be considered. This is, of course, impracticable since the endurance for any specimen under given loading cannot be predicted. The practice of applying a factor of 1·25 to the alternating load, however, and the use of the logarithmic instead of the arithmetic mean goes some way towards removing errors and inconsistencies.

This factor of safety applied to alternating load is the main reason why a value as high as two-thirds the mean is taken for the probable minimum. It has been calculated* that this value corresponds to 10 per cent. of a large number of nominally identical specimens having a shorter life under the loading system applied in the tests. It is considered, however, that in all reasonable circumstances it gives a safe minimum for *all specimens* under the unfactored loading of the basic condition.

It is thus seen that the factor on alternating load and the factor for variation are considered together, and the final result is purely empirical and based on experience. If a very large number of tests could be made, more accurate statistical methods might be feasible for determining the probable minimum. For ordinary routine work, however, a large number of tests is impracticable and, in any case, the accuracy of the formula for life and of the data used are not such as to warrant attempts at more accurate determination of the safe minimum. To avoid misunderstanding, the value for the safe minimum calculated in the somewhat arbitrary way described is referred to as the " Nominal Endurance."

There is one further complication. Since at least six tests are required, the testing of complete wing systems for direct life assessment is beyond the resources normally available. Tests have therefore to be made on a number of components, *e.g.* joints, selected as likely to be critical. The smallest value of Nominal Endurance obtained for these is then taken as the value for the wing system as a whole.

There is thus a danger of an incorrect life assessment being obtained through some critical component being overlooked. Great care is necessary, therefore, in selecting components for test; and it is desirable to err deliberately on the safe side by including any that are suspect, however slightly. In this connection a single test on a complete wing system is of great value and, although costly, is usually necessary. To allow for variation, the test is carried well beyond the initial failure. When a component breaks it is repaired and the test is continued. Further repairs are made as necessary until, after allowance has been made for reasonable variation, it is reasonably certain that the weakest component is included in the list of potentially weak components. All these are then tested separately with the required number of repetition tests.

The whole procedure for determining the Nominal Endurance for final life estimation may now be summarised as follows.

(*a*) The potentially weak components are selected, preferably with the help of a test on a complete wing system.

(*b*) Each of these components is tested at least six times to give values for endurance n_1, n_2, n_3, and so on.

(*c*) The logarithmic mean (M) of n_1, n_2, n_3, and so on, is then determined.

(*d*) The Nominal Endurance (N), or safe minimum, is then calculated as $2M/3$.

(*e*) The lowest value of Standard Life for all the components is taken to be the Standard Life for the complete wing system.

5. The Final Formula—" Standard Life "

To obtain the formula for life it is necessary to review the main assumptions. The aircraft is assumed

*By J. Taylor of the R.A.E.

to be flying at its normal operating speed, and to encounter a series of pairs of alternate up and down gusts of velocity 8 equivalent feet per second. This simplified condition is assumed to produce the same effects as the complicated pattern of fluctuating load encountered in flight.

To complete the simplified picture, however, a numerical quantity is necessary, representing the frequency with which the hypothetical gust cycles are encountered. This can be expressed in terms of the number of simplified gust cycles per mile, but in practice it is more convenient to use the reciprocal, *i.e.* the average distance (m miles) between the beginning of one cycle and beginning of the next.

On this basis the safe life in miles (M) may be expressed by the formula

$$M = kmN$$

where N is the nominal test endurance, m is the distance of separation in the idealised gust picture just described, and k is some overall factor which may be regarded as a factor of safety.

The merit in using the idealised gust picture is that it reduces to a common basis the external conditions affecting aircraft that may differ in their responses to those conditions. Thus, in calculation of the alternating load for the fatigue tests, the characteristics of the particular aircraft, *e.g.* wing loading, operating speed and aerodynamic features, are taken into account directly and specifically. Furthermore, it gives the frequency with which gusts are encountered in terms of distance travelled, an essential requirement since gusts are disposed geographically and not related to the speed of the particular aircraft.

At this stage, however, the idealised gust picture has served its purpose and can be abandoned. It does not help in any direct way in determining the empirical constant used in the formula. The value distance of separation (m) is not known and, moreover, cannot be separated from the factor of safety. The product (km), in fact, is a single empirical factor to be chosen in the light of experience.

Since life is normally expressed in flying hours the next step is conversion to a time basis. This is a simple matter when the average true speed is known. There are, however, advantages in working in terms of equivalent speed in preference to true speed. In particular, equivalent speed is used in defining the alternating load for the fatigue test, and it is useful to have the same variable serve the two purposes. For aircraft operating below 15,000 ft. the error that could be introduced is small and within the tolerance allowed on an empirical formula, especially since the overall empirical constant can be adjusted with this change in mind. The formula for safe life (L) in flying hours thus takes the form

$$L = KN/V$$

where K is a new constant and V is the normal operating equivalent speed, which may be expressed conventionally in knots.

In actual fact, equivalent speed gives a more reliable formula than does true speed for reasons not yet considered. As operating height increases, the frequency of gusts diminishes and this is reflected in some degree in the formula by the reduction of equivalent speed for a given true speed. The effect of operational height is responsible for one of the most important reservations in the use of the formula, as discussed later. The reservation still remains when equivalent speed is used instead of true speed, but is rather less significant. Thus while operators using the formula do not obtain the full benefit of high flying, they are at least not penalised for it.

The final stage is to determine the value of the empirical constant (K) to give the "Standard Life." The value chosen in the light of all the experience available is 2·5. This is deemed to correspond to reasonably good operating conditions, actually those normally occurring on European air lines. The final formula, therefore becomes:—

$$L = 2{\cdot}5\ N/V$$

where

L is the "Standard Life" in flying hours;

N is the "Nominal Test Endurance";

V is the equivalent operating speed in knots.

It is of interest to remark that the life obtained from this formula is less than that corresponding to the criterion previously suggested by the author[3] with reservations concerning future empirical adjustment as more information was obtained. The main cause of the reduction in estimated safe life is the greater allowance that is now found to be necessary for variation of nominally identical specimens.

6. Application and Reservations

It is now proposed to consider the use of the formula and the reservations which must be made. There are many circumstances which may render the formula inaccurate. Of these, the operating height above the ground is usually the most significant.

The formula as given is intended to apply to aircraft which normally operate above 8,000 ft. For aircraft flying below this height a shorter life is to be expected owing to the greater turbulence of the lower regions. For aircraft flying at about 5,000 ft. the indications are that not more than half the Standard Life can safely be assumed, with some intermediate correcting factor for flight between 5,000 ft. and 8,000 ft. Below 5,000 ft. still shorter life can be expected, and in extreme cases less than one-sixth of the "Standard Life" is possible.

Length of journeys, *i.e.* distance between the take-off and landing, is also a factor to be considered for aircraft that normally fly high. However high the aircraft may fly, it has only to ascend and descend once per journey through the gusty lower levels of the atmosphere. The "Standard Life" may therefore be unduly low as an estimate of actual life for high flying aircraft where journeys are long. The situation is complicated by the indirect relationship between operating height and length of journey, since aircraft operating at great heights are unlikely to be used for short journeys.

The geographical region is another factor influencing life. Gust frequencies and intensities vary in different

parts of the world. It is now considered, however, that this variation is less than was at one time supposed. Until recently, for example, it was thought that conditions in Central Africa were much more severe than in Europe. The latest evidence indicates that the shorter life does in fact exist but is attributable largely to the lower operating height. In this connection it is important to bear in mind that it is height above the ground, and not height in standard atmosphere, that normally governs the gust distribution. Hence a high ground level encourages still further the tendency to fly at a fairly low height above the ground for journeys of short or even moderate length.

The fatigue life of an aircraft can also be influenced by the pilot, or by operators responsible for briefing him. Care in avoiding gusty regions, and deliberate slowing down when they are unavoidable, can increase fatigue life considerably. For aircraft flying at more than 5,000 ft., moreover, considerable saving in life can be achieved by reducing speed in climb and descent. A further saving is possible by climbing and descending at a steep angle, so reducing the time spent in the gusty lower regions. Loitering while waiting to land, on the other hand, reduces the life of the aircraft.

All the circumstances so far mentioned introduce errors that arise not so much from faults in basic treatment as from the broad generalisations necessary for simplicity. It is possible to obtain closer estimates by adjustment of the formula to meet the special conditions. There are prospects, moreover, of a rather more elaborate formula being developed to cover more specifically the particular operating conditions that are at present treated in a broad general way. Such elaboration verges on the more exact methods often used in the later stages in the life of an aircraft when statistical operational data have been collected.

There are other circumstances which cannot be treated by simple correction or adaptation of the formula. Most important is the effect of short journeys considered without regard to any associated effects of low operating height. For very short journeys, landing, taxying and take-off may seriously reduce the safe life beyond anything contemplated for the simple formula. It is possible for journeys to be so short that even the loading and unloading of the wings, as occurring once each flight, becomes significant. Conditions such as these are deemed to be abnormal and to require special investigation.

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The Fluid Mechanics of Detonation Waves

by

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1. Introduction

In a previous paper [1] it has been shown that a general method may be developed for calculating the various changes which take place in a flowing gas. The changes considered were the addition and extraction of heat in a subsonic flow, the addition and extraction of heat in a supersonic flow and sudden changes, such as shock waves. The changes were considered to be stationary and to take place in a parallel pipe. It was of interest to see if a similar approach would produce a general treatment for moving waves. The investigation was limited to moving shock waves and detonation waves.

The most interesting travelling discontinuity is an intense detonation wave which is liberating a large quantity of heat. For this type of wave it does not appear to be possible to produce a treatment similar to that suggested for stationary waves. However, the investigation required a clear understanding of the relationship between detonation waves and simple travelling shock waves. The established literature on the subject of detonation waves appears to have been written long before these discontinuous processes were of interest to the aeronautical engineer. It is the author's opinion that the orthodox treatment of detonation waves tends to make the subject far more difficult than it need be.

The orthodox method sets out to calculate an equation which gives the relation between the velocity of a detonation wave, and the velocity of the gas behind the wave, for a fixed heat release. The equation is produced as a result of using three equations:—

(1) Conservation of mass flow,

(2) An equation involving change of momentum,

(3) An equation relating the total energies before and after the change.

These equations provide a relation between the wave speed and the velocity of the gas after the wave, and no more. It is usually suggested that there are not sufficient equations to define the problem and an additional assumption is necessary. It will be shown that the information available is the same as in most flow problems. The quantities which have to be fixed to define a detonation wave are similar to those which have to be decided in a simple flow, such as the discharge of a gas from a reservoir. The present paper is an attempt to give a clear statement of the way in which the equations of conservation of flow, momentum and energy apply to moving shock waves and detonation waves. It points out the relation between these types of wave and shows that the confusion in the orthodox treatment has arisen simply because the equations have been used in a way which disguised the physics of the problem.

Originally received February 1952.

Notation

A	speed of sound corresponding to total temperature T
a	speed of sound corresponding to static temperature
a_3	speed of sound in gas after expansion to pressure P_3 (Fig. 5)
I	impulse defined as $p+\rho u^2$
P	gas pressure
P_3	pressure in expansion process (Fig. 5)
R	gas constant
J	mechanical equivalent of heat
T	total temperature of gas
ΔT	change in gas total temperature
t	static temperature of gas
u	wave speed
Δw	change in gas speed through expansion wave
w	speed of gas following wave, or speed of piston
W	gas mass flow
ρ	gas density
γ	ratio of specific heats
K_p	specific heat at constant pressure
K_v	specific heat at constant volume

Suffix 1 refers to conditions before wave.
Suffix 2 refers to conditions after wave.

2. Discussion of Established Treatment of Detonation Waves

The first step in understanding the problem is to follow the established treatment and derive the equation which relates the wave speed to the speed of the gas following it. The methods of Ref. 1 will be used, because they provide a simple way of handling the equations of conservation of flow, momentum, and changes in energy. The method used is similar to the orthodox treatment. The equations are written down for the stationary wave, and the velocities calculated before and after the wave. The stationary wave system is then given a forward velocity equal to the wave speed and the velocities in the moving system are deduced from those found for the stationary case.

Equation (1) in Appendix I gives the relation between the velocities before and behind a stationary wave.

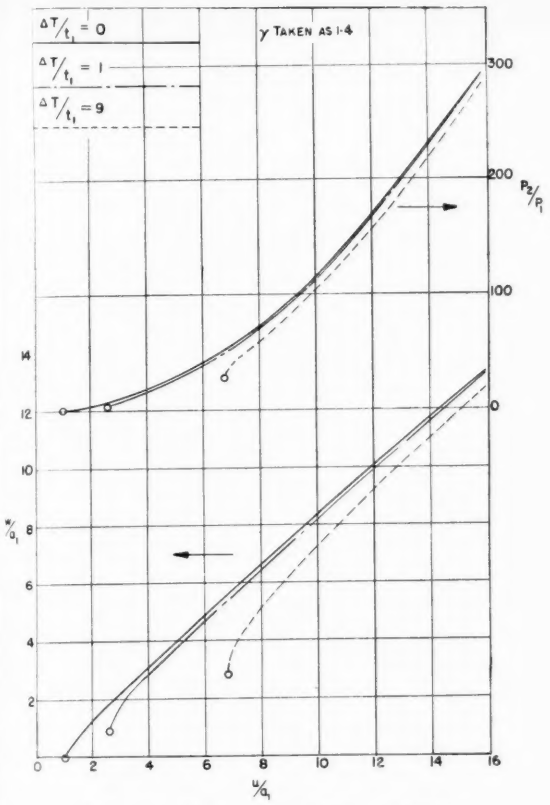


FIGURE 1. Moving waves with heat addition.

This Appendix outlines the method of translating this equation so that it applies to the moving wave. Equation (4) defines the wave speed in terms of the velocity following it, and the heat released. To aid discussion this equation is reproduced here.

$$\frac{w}{a_1} = \frac{\left(\frac{u}{a_1}\right)^2 - 1}{\left|\frac{u}{a_1}\right|} \frac{1}{\gamma+1}\left[1 \pm \sqrt{\left\{1 - \frac{\left(\gamma+1\right)\frac{\Delta T}{t_1} 2\left(\frac{u}{a_1}\right)^2}{\left[\left(\frac{u}{a_1}\right)^2 - 1\right]^2}\right\}}\right]$$

The equation contains three variables: the speed of the wave, the speed of the gas following the wave and the heat released. The speeds are written non-dimensionally in that they are divided by the speed of sound in the stationary gas in front of the wave.

The orthodox treatment says that additional information must be given to define the wave speed. This equation will not give the answer by itself. To illustrate this, Fig. 1 shows the speed of the gas following the wave in terms of the wave speed. The static pressure ratio across the wave is also plotted. In drawing these curves a particular solution has been chosen. The solution chosen is for the case when the wave is moving faster than the speed of sound in the stationary fluid in front of it, and the speed of the gas following the wave is less than the local speed of sound. When no heat is released this corresponds to a travelling shock wave and one of the curves in Fig. 1 is drawn for this case. Experimentally it has been shown that a wave in which heat is released will travel with a speed greater than the speed of sound in the fluid before it, and that it travels with a particular velocity depending on the amount of heat released. The speed with which the wave travels in practice corresponds to the first point at which it is possible to obtain a solution to the equation. These points are marked with a circle in Fig. 1.

There is considerable discussion in the established literature to show that the wave ought to travel at this speed because of something to do with entropy. The author is never sure what these discussions mean, and thinks the alternative explanation a much better one. In Fig. 2, the sum of the speed of the gas following the wave and the speed of sound after the wave, is plotted against the wave speed. For convenience, the sum, of the speed of gas following the wave and the local speed of sound, has been divided by the speed of the wave, and this quantity is plotted against wave speed expressed non-dimensionally. When the quantity plotted on the vertical scale is more than one a disturbance in the gas behind the wave will overtake the wave, but when it is less than one it will not. Physical considerations suggest that in practice the wave travels at a speed where the disturbances just fail to overtake it. In other words, the sum of the speed of the gas following the wave and the local speed of sound is equal to the wave speed. This point is marked with a circle and is the point where it is first possible to obtain a solution to the equation. When the argument is approached as it has been so far it is plausible, but a little difficult to understand. However, when the problem is stated properly and the methods of the next section are used to derive the expression for the wave speed, the physics of the process becomes clear and the present difficulties do not arise.

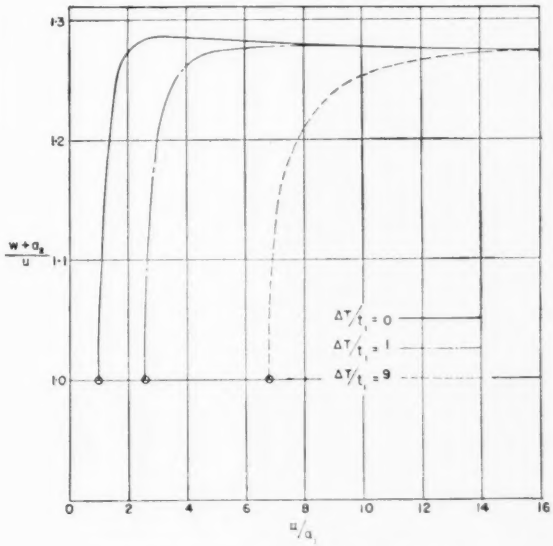


FIGURE 2. Moving waves with heat addition.

3. Derivation of the Relations for the Detonation Wave from First Principles

The whole difficulty in the established treatment of the problem is that it is dealing with a system which is not steady. It does not describe what happens to the gas some distance behind the wave. To produce a system which does not change with time, it is necessary to consider that a piston is following the wave. Imagining a system in which a piston is moved down a tube with sufficient speed, the speed of the piston will define the speed of the gas following the wave, and hence the wave speed. This system may be considered a steady system, because the speed of the gas between the face of the piston and the moving wave is constant. As the wave moves through the undisturbed gas, it changes the gas from one condition which is constant with time, to another condition which is constant with time.

The problem of calculating the velocity of a wave which is travelling ahead of a moving piston is now considered. This wave may be moved through a combustible mixture and its passage may release heat. In this case it is a detonation wave. The mathematics of the system are set out in Appendix II. The momentum and continuity equations are simple to derive. In writing down the energy equation it must be remembered that the piston is doing mechanical work. There is a pressure acting on the face of the piston and it is moving with a velocity w. If the piston is of unit area then the work done is $P_2 w$ mechanical units per second. For convenience the equation is written in terms of a unit mass of gas. The mass of gas encompassed by the wave per second is $\rho_1 u$, so that the work done by the piston per unit mass of gas per second is $P_2 w/(\rho_1 u)$. The heat release per unit mass of gas is known. The sum of this and the work done by the piston is equal to the increase in internal energy of the gas plus the kinetic energy after the wave. A little algebra produces the same equation as that derived in Appendix I, this equation being the relation between the speed of the gas following the wave and the wave speed, if the heat release is known. This equation now has a perfectly clear physical interpretation. For every speed of the piston there is a particular wave speed, provided that the heat release is known. The pressure ratio across the wave may be derived very simply from the momentum equation (*see* equation (9) in Appendix II). By choosing the speed of the piston, the pressure ratio across the wave is decided. There is a certain wave speed corresponding to a certain pressure ratio. This is analogous to the discharge from a reservoir through a nozzle. The velocity at the throat of the nozzle may be written in terms of the ratio of the reservoir pressure to atmospheric pressure. The equation for this system is

$$\frac{P_1}{P_2} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}},$$

where P_1 is the reservoir pressure, P_2 the atmospheric pressure and M the Mach number in the throat of the

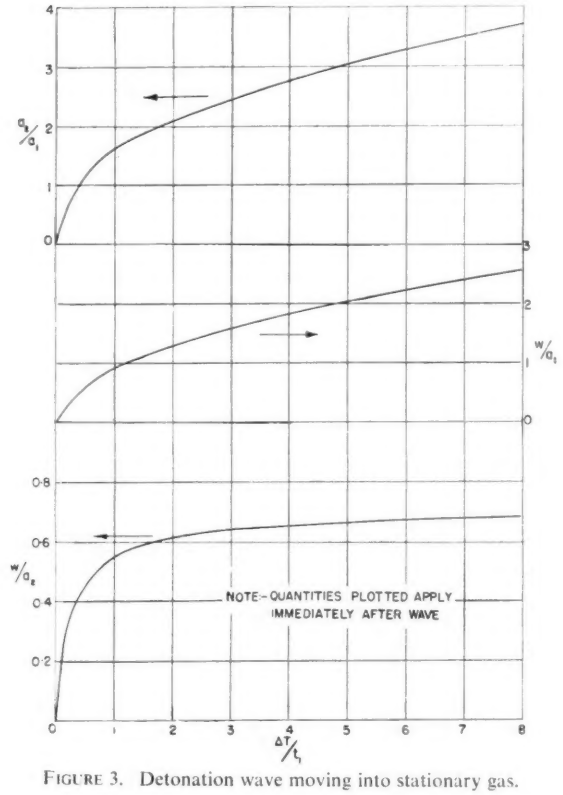


FIGURE 3. Detonation wave moving into stationary gas.

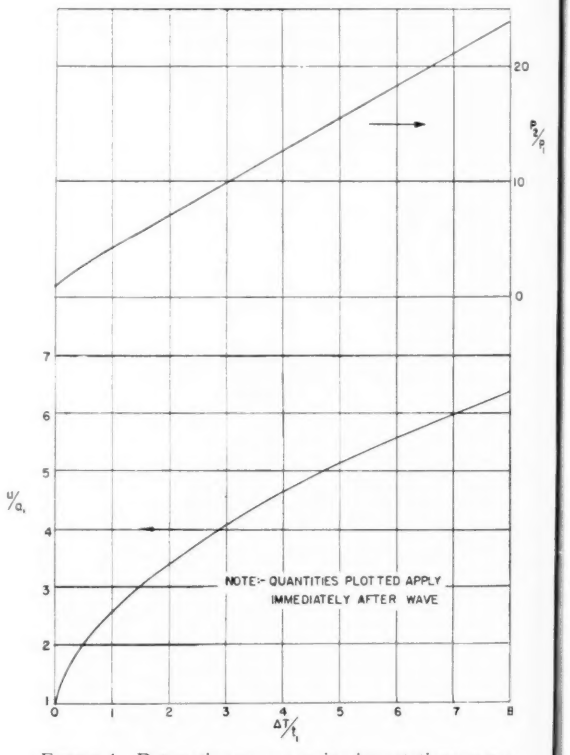


FIGURE 4. Detonation wave moving into stationary gas.

nozzle. In this equation the Mach number of the discharge cannot be determined until the pressure ratio is chosen. The travelling wave, be it a simple shock wave or a detonation wave which is releasing heat, is exactly similar. Its speed is not known until the pressure ratio across the wave is stated.

The difficulties in the established treatment are seen to be the result of not considering the whole system. As soon as the piston is introduced into the argument and the equation is written down from that point of view, the physics of the process is fairly clear.

An interesting point is that the normal method, which derives the results from the stationary wave case, unwittingly goes through the same process. Appendix III examines the transformation of the energy equation from the stationary wave to the moving wave case. This transformation demonstrates that for the two forms of the energy equation to be consistent, a term which is the work done by the piston must appear in the moving wave case.

4. Detonation Waves in Practice

There have been many experiments on detonation waves in tubes, and these waves are produced without the aid of pistons or other mechanical devices. The best way to visualise the speed at which a detonation wave will travel, without the aid of a piston, is to imagine that the wave is started by the piston and then the piston is stopped. Assuming it were possible to produce a piston which would move with the required velocities, there is no doubt that detonation waves could be generated which would travel with any speed. The speed would be decided by the speed of the piston. This means that for a given heat release any point on the appropriate curve in Fig. 1 or Fig. 2 is possible. If the piston is stopped an expansion wave will pursue the detonation wave and will lower the pressure behind it. The pressure behind the moving wave will be reduced until no more expansion waves can overtake the detonation wave. At this point the wave speed will be equal to the sum of the speed of the gas immediately behind the wave and the local speed of sound. This is the point at which it is first possible to obtain a solution to the velocity equation which is derived in Appendices I and II. Choosing this particular solution of the equations it is possible to plot the changes which occur across a detonation wave, for varying heat addition, and these results are shown in Figs. 3 and 4.

5. Structure of Detonation Wave

It is clear that any practical detonation wave, which is one which is not followed by a piston, is an amalgamation of a compression wave and an expansion wave, and it is interesting to examine the structure of the wave in detail.

A simple expansion wave may be produced by withdrawing a piston from a tube in which the gas starts from rest. The gas adjacent to the head of the piston will move in the same direction at the same speed. The head of the expansion wave advances into the undisturbed gas in a direction opposite to that of the piston and moves with the local speed of sound. The expressions for the change in velocity through an expansion wave are derived in Appendix IV, and examination of these will show that the changing velocity through an expansion wave, for a given ratio of static pressures

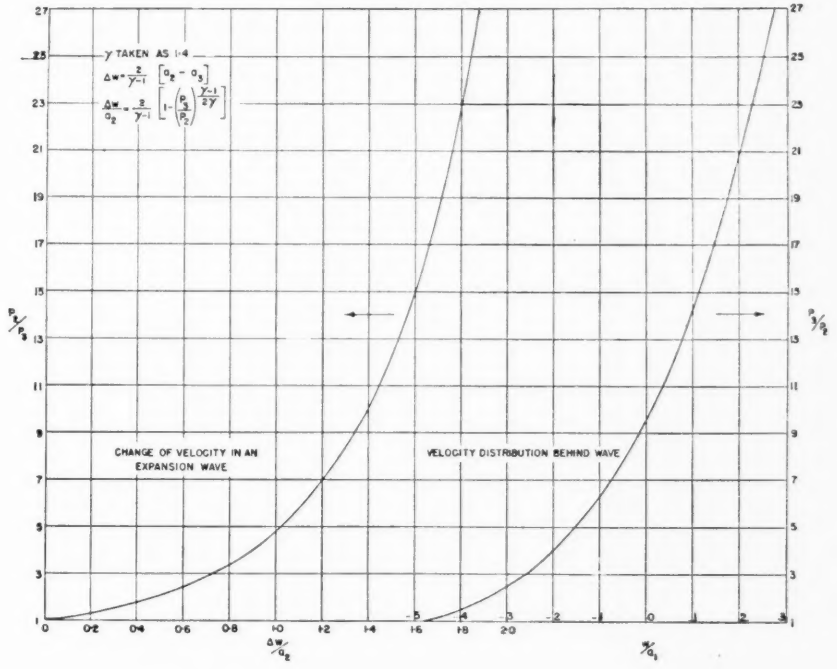


FIGURE 5.

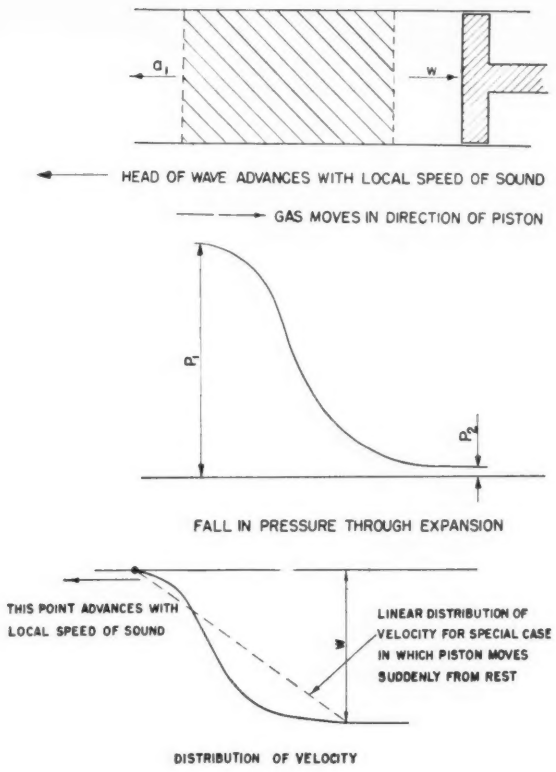

a_1
w
HEAD OF WAVE ADVANCES WITH LOCAL SPEED OF SOUND
GAS MOVES IN DIRECTION OF PISTON
p_1
p_2
FALL IN PRESSURE THROUGH EXPANSION
THIS POINT ADVANCES WITH LOCAL SPEED OF SOUND
w
LINEAR DISTRIBUTION OF VELOCITY FOR SPECIAL CASE IN WHICH PISTON MOVES SUDDENLY FROM REST
DISTRIBUTION OF VELOCITY

FIGURE 6. Simple expansion wave.

before and after the wave, is independent of the general gas speed.

Sir Geoffrey Taylor in a paper on Detonation, the full title and reference of which are given in Appendix IV, has shown that if the piston were started from rest and achieved its steady velocity instantaneously, then the velocity distribution would be linear through the expansion wave. This is also true for a piston moving steadily in one direction and suddenly changing its velocity to some chosen value in the opposite direction.

The detonation wave, when followed by a piston, consists of a sudden rise in pressure and change in velocity as shown in Fig. 7. Conditions between the wave front and the head of the piston are uniform. To understand the practical detonation wave which is closely followed by an expansion wave, we may imagine the following processes. The wave is set going by a piston moving in the same direction, then the piston is suddenly reversed to some chosen velocity in the opposite direction. This generates an expansion wave which overtakes the detonation wave. The final state is a detonation wave followed by an expansion wave with a linear velocity distribution through it. The fact that with the assumptions made the velocity distribution in the expansion wave following the detonation wave is linear, was first pointed out by Sir Geoffrey Taylor in the paper already referred to.

One can imagine several types of expansion processes following the wave. The system might have been set going by a piston and then the velocity of the piston suddenly reduced to a lower velocity. In this case the expansion wave would be of the strength required to slow the gas down to the velocity of the piston. In another case the piston might set the wave going and then suddenly come to rest. This would correspond to detonation in a tube with a closed end. Finally the piston might set the wave in motion and then be reversed. Here there is a restriction on the velocity of the gas as it flows away. This velocity may not be greater than the local speed of sound in the gas, providing the processes are taking place in a parallel tube. In a practical case, the pressure ratio across the detonation wave is sufficiently high that the flow of the gas down the tube would almost certainly be restricted by the condition that it cannot be greater than the local speed of sound. This would correspond to detonation starting at the open end of a tube. Pressure ratios produced by practical detonation waves are high, and as an example, a wave with a pressure ratio of 27 across it is considered. In terms of the velocity of sound in front of the detonation wave, the gas immediately behind the wave is moving in the same direction with a velocity of 2·8. The expansion wave reduces this velocity to zero

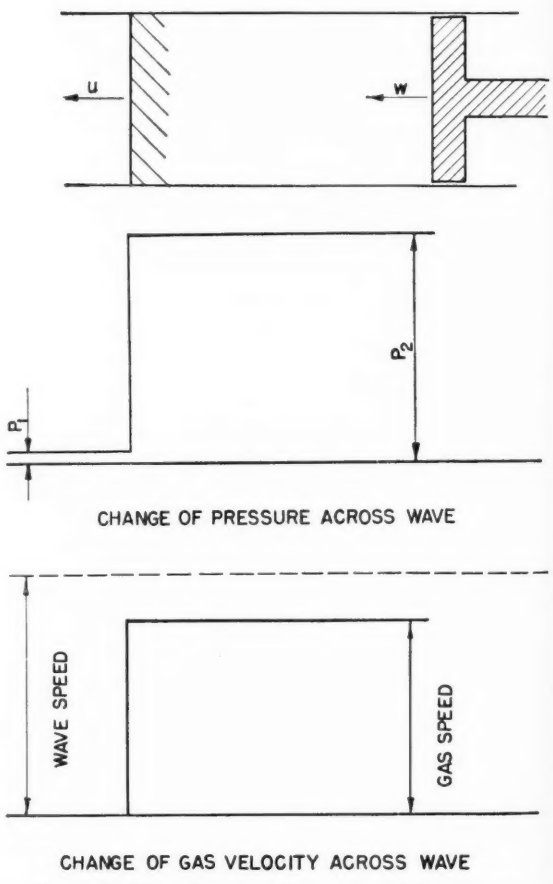


FIGURE 7. Normal compression wave or detonation wave stabilised by piston.

when the static pressure is still 9·5 times the static pressure in front of the wave. As the pressure is reduced still more the gas flows in the opposite direction until it reaches the local speed of sound, when it continues to flow with a constant velocity until it reaches the open end. With the assumed value of γ the pressure ratio required to expand the gas from the condition of zero velocity to the state where the flow velocity is equal to the local speed of sound is 3·58, thus the burnt mixture reaches the open end of the tube with a pressure of 2·65 times the static pressure in front of the detonation wave. The velocities of the gas behind the wave have been plotted against the pressure ratio in Fig. 5 and a diagram of the system is shown in Fig. 8.

6. Assumptions

A final consideration is the assumptions which have been made in deriving the expressions for wave speed. In all the cases dealing with compression waves, it has been assumed that the rise in pressure is instantaneous. The expression given in Appendix II for the work done by the piston per unit mass of gas encompassed by the wave each second is only true for sudden compression. It is not true for an expansion wave. It has also been assumed that combustion takes place in an extremely narrow band in the wave front. It is found that the structure of the wave must consist of a sudden compression followed by an expansion, the width of the expansion region in which there is a high pressure being considerable. If this is so, it is unlikely that all the combustion occurs in the wave front. In one of his many tests on detonation, Bone succeeded in firing the same mixture four times and producing a detonation each time. Obviously combustion was far from complete in any one detonation wave. Apart from these assumptions, the usual assumptions have been made as to the validity of gas laws and the constancy of γ. These latter simplifying assumptions enable the equations to be derived to illustrate the mechanism of the processes. The calculations on an actual detonation wave are much more complicated.

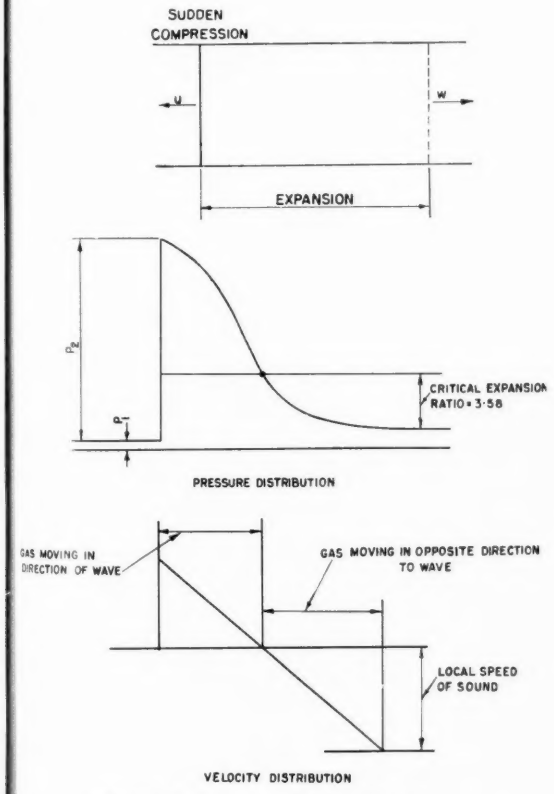


FIGURE 8. Structure of detonation wave.

APPENDIX I

MOVING WAVE RELATIONS DEDUCED FROM THE STATIONARY CASE

In a stationary wave the expression $(W\sqrt{T}/I)\sqrt{(R/\gamma)}$ remains constant. In problems involving heat addition only $(W/I)\sqrt{(R/\gamma)}$ is constant.

$$\frac{W\sqrt{T}}{I}\sqrt{\left(\frac{R}{\gamma}\right)} = \frac{WA}{I}\frac{1}{\gamma},$$

where A is the speed of sound corresponding to the temperature T.

It has been demonstrated that

$$\frac{W\sqrt{T}}{I}\sqrt{\left(\frac{R}{\gamma}\right)} = \frac{u}{A}\frac{1}{1+\frac{\gamma+1}{2}\left(\frac{u}{A}\right)^2},$$

and hence

$$\frac{W}{I}\frac{1}{\gamma} = \frac{u}{A^2}\frac{1}{1+\frac{\gamma+1}{2}\left(\frac{u}{A}\right)^2}.$$

If u_1 and u_2 are the velocities before and after the stationary wave and A_1, A_2 are the speeds of sound corresponding to the total temperatures, then the fact that W/I is constant gives the equation

$$\frac{u_1}{A_1^2}\frac{1}{1+\frac{\gamma+1}{2}\left(\frac{u_1}{A_1}\right)^2} = \frac{u_2}{A_2^2}\frac{1}{1+\frac{\gamma+1}{2}\left(\frac{u_2}{A_2}\right)^2} \qquad (1)$$

This equation may be written

$$u_1A_2^2 - u_2A_1^2 + \frac{\gamma+1}{2}u_1u_2[u_2-u_1] = 0,$$

and substituting $w = u_1 - u_2$ it becomes

$$u_1(A_2^2 - A_1^2) + wA_1^2 - \frac{\gamma+1}{2}w(u_1-w)u_1 = 0. \qquad (2)$$

When this equation is applied to the moving wave case the term $(A_2^2 - A_1^2)$ may be carried over. For the

stationary wave this term was proportional to the increase in total temperature ΔT,* and the same applies to the moving wave.

For convenience the equation may be divided by a_1^2 where a_1 is the speed of sound corresponding to the static temperature t_1 in front of the moving wave.

Remembering that the energy equation

$$K_p T_1 = K_p t_1 + \tfrac{1}{2} u_1^2$$

may be written

$$\frac{T_1}{t_1} = 1 + \frac{\gamma-1}{2}\left(\frac{u_1}{a_1}\right)^2,$$

since $a_1^2 = \gamma R t_1$, and using the fact that

$$\frac{T_1}{t_1} = \left(\frac{A_1}{a_1}\right)^2$$

gives $$\left(\frac{A_1}{a_1}\right)^2 = 1 + \frac{\gamma-1}{2}\left(\frac{u_1}{a_1}\right)^2.$$

Then equation (2) may be written

$$u_1 \frac{\Delta T}{t_1} + w\left(\frac{A_1}{a_1}\right)^2 - \frac{\gamma+1}{2}\frac{w u_1 (u_1 - w)}{a_1^2} = 0$$

or

$$u_1 \frac{\Delta T}{t_1} + w\left[1 + \frac{\gamma-1}{2}\left(\frac{u_1}{a_1}\right)^2\right] - \frac{\gamma+1}{2}\frac{w u_1 (u_1 - w)}{a_1^2} = 0.$$

Applying the equation to the moving wave, and dropping the suffix from u_1, the equation reduces to

$$\left(\frac{w}{a_1}\right)^2 - \frac{2}{\gamma+1}\left(\frac{w}{a_1}\right)\frac{\left[\left(\frac{u}{a_1}\right)^2 - 1\right]}{\frac{u}{a_1}} + \frac{2}{\gamma+1}\frac{\Delta T}{t_1} = 0 \qquad (3)$$

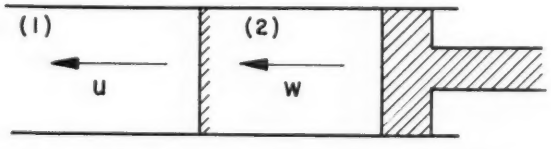
where w is the speed of the gas following the wave, and u is the wave speed.

The solution to this is

$$\frac{w}{a_1} = \frac{\left(\frac{u}{a_1}\right)^2 - 1}{\frac{u}{a_1}} \frac{1}{\gamma+1}\left[1 \pm \sqrt{\left\{1 - \frac{2(\gamma+1)\frac{\Delta T}{t_1}\left(\frac{u}{a_1}\right)^2}{\left[\left(\frac{u}{a_1}\right)^2 - 1\right]^2}\right\}}\right] \qquad (4)$$

APPENDIX II

FUNDAMENTAL DERIVATION OF MOVING WAVE EQUATIONS



As is usual the three equations of momentum, continuity and energy must be written down.

The momentum equation is

$$P_2 - P_1 = \rho_1 u w \quad . \quad . \quad . \qquad (5)$$

The continuity equation is

$$\rho_2 w = (\rho - \rho_1) u \quad . \quad . \quad . \qquad (6)$$

(It is a help to derive this from the stationary wave case).

The energy equation cannot be written down, without imagining a piston following the wave; the velocity of the piston is w, and is the same as the velocity of the gas following the wave.

There are the following terms to consider:—

The work done by the piston.
The heat liberated by combustion in the wave front.
The increase in kinetic energy of the gas.
The increase in internal energy of the gas.

The work done by the piston per unit mass of gas encompassed by the wave each second is $P_2 w/(\rho_1 u)$.

The other terms yield the equation

$$\frac{P_2 w}{\rho_1 u} + J K_p \Delta T = J K_v (t_2 - t_1) + \tfrac{1}{2} w^2,$$

which may be written

$$\frac{P_2}{P_1}\frac{w}{u} + \frac{\gamma}{\gamma-1}\frac{\Delta T}{t_1} = \frac{1}{\gamma-1}\frac{t_2 - t_1}{t_1} + \frac{\gamma}{2}\left(\frac{w}{a_1}\right)^2 \qquad (7)$$

after taking a_1 the speed of sound in front of the wave as $a_1 = \sqrt{(\gamma R t_1)}$,

and using $$\frac{R}{J} = K_p - K_v$$

and $$\frac{P_1}{\rho_1} = R t_1.$$

Equations (5), (6) and (7) combine to give

$$\left(\frac{w}{a_1}\right)^2 - \frac{2}{\gamma+1} \cdot \frac{w}{a_1} \cdot \frac{\left(\frac{u}{a_1}\right)^2 - 1}{\frac{u}{a_1}} + \frac{2}{\gamma+1}\frac{\Delta T}{t_1} = 0 \qquad (8)$$

which is the same equation as in Appendix I.

For the derivation of pressure ratio, the momentum equation gives

$$P_2 - P_1 = \rho_1 u w.$$

This may be written

$$\frac{P_2}{P_1} - 1 = \frac{\gamma u w}{\gamma P_1/\rho_1},$$

which gives

$$\frac{P_2}{P_1} = 1 + \frac{\gamma u w}{a_1^2}, \quad . \quad . \quad . \qquad (9)$$

since $a_1 = \sqrt{(\gamma P_1/\rho_1)}$ = speed of sound before the wave.

APPENDIX III

THE RELATION BETWEEN THE MOVING AND STATIONARY WAVE CASES

Appendix I demonstrates a quick way of handling the equations of momentum, continuity and energy, based on previous work, for the stationary wave.

*See Note on page 626.

Had the equations been written in full they would have been

Momentum: $P_1+\rho_1 u_1^2=P_2+\rho_2 u_2^2$,

Continuity: $\rho_1 u_1=\rho_2 u_2$,

Energy:

$$\frac{P_1}{\rho_1}+JK_v t_1+\tfrac{1}{2}u_1^2+JK_p\Delta T=\frac{P_2}{\rho_2}+JK_v t_2+\tfrac{1}{2}u_2^2.$$

These equations are made to apply to the moving wave case by making the substitution $u_2=u_1-w$.

For the momentum and continuity equations the forms given in Appendix II are obtained very easily. The energy equation is not so straightforward.

Making the substitution $u_2=u_1-w$ gives

$$\frac{P_1}{\rho_1}+JK_v t_1+JK_p\Delta T=\frac{P_2}{\rho_2}+JK_v t_2+\tfrac{1}{2}w^2-u_1 w,$$

and re-writing,

$$\frac{P_1}{\rho_1}-\frac{P_2}{\rho_2}+u_1 w+JK_p\Delta T=JK_v(t_2-t_1)+\tfrac{1}{2}w^2.$$

Consider the term

$$\frac{P_1}{\rho_1}-\frac{P_2}{\rho_2}+u_1 w.$$

From the momentum equation

$$\rho_1 u_1 w=P_2-P_1,$$

$$\frac{P_1}{\rho_1}-\frac{P_2}{\rho_2}+u_1 w=\frac{P_1}{\rho_1}-\frac{P_2}{\rho_2}+\frac{P_2-P_1}{\rho_1}$$

$$=P_2\left[\frac{1}{\rho_1}-\frac{1}{\rho_2}\right]$$

or

$$=P_2\,\frac{\rho_2-\rho_1}{\rho_1\rho_2}.$$

But continuity gives

$$\frac{\rho_2-\rho_1}{\rho_2}=\frac{w}{u_1}$$

and hence

$$\frac{P_1}{\rho_1}-\frac{P_2}{\rho_2}+u_1 w=\frac{P_2 w}{\rho_1 u_1},$$

and represents the work done by the piston per unit mass of gas encompassed by the wave per second.

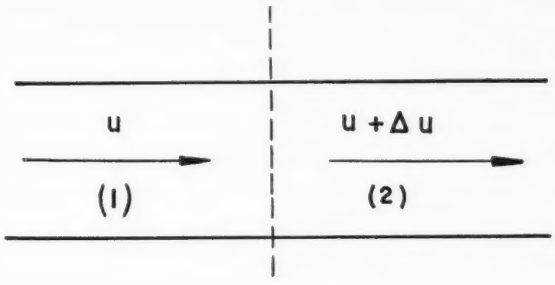
This relationship for the energy equation having been established, it appears that the energy equation written down in Appendix II from physical considerations, is identical with that obtained by manipulating the equation for the stationary wave case.

APPENDIX IV

EXPANSION WAVES

The changes across an elementary stationary wave are given by the momentum equation,

$$\Delta P=\rho u\,\Delta u$$



and by continuity

$$\frac{\Delta u}{u}=-\frac{\Delta\rho}{\rho}.$$

Combining these gives in the limit

$$\frac{dp}{d\rho}=u^2=a^2=(\text{speed of sound})^2.$$

The changes across an expansion wave are isentropic, and thus

$$\frac{p}{\rho\gamma}=\text{a constant}.$$

For this case

$$\frac{dp}{d\rho}=\frac{\gamma p}{\rho},$$

and thus $a^2=\dfrac{\gamma\rho}{\rho}$.

Differentiation gives

$$2a\,da=\frac{\gamma dp}{\rho}-\frac{\gamma p}{\rho}\frac{d\rho}{\rho}$$

$$=\frac{\gamma dp}{d\rho}\frac{d\rho}{\rho}-\frac{\gamma p}{\rho}\frac{d\rho}{\rho},$$

giving

$$a\,da=\frac{\gamma-1}{2}a^2\frac{d\rho}{\rho}.$$

But

$$\frac{dp}{\rho}=-\frac{du}{a},$$

and therefore

$$da=-\frac{\gamma-1}{2}du.$$

Integration gives

$$a_1-a_2=\frac{\gamma-1}{2}(u_2-u_1).$$

For a wave moving into a stationary fluid, the change in velocity due to the wave is given by

$$\Delta w=\frac{2}{\gamma-1}(a_1-a_2),$$

and is in the opposite direction to the wave motion.

The pressure ratio across the expansion wave may be found by writing

$$\Delta w=\frac{2}{\gamma-1}(a_1-a_2)$$

as

$$\frac{\Delta w}{a_1}=\frac{2}{\gamma-1}\left[1-\frac{a_2}{a_1}\right]=\frac{2}{\gamma-1}\left[1-\frac{P_2}{P_1}\,\frac{\rho_1}{\rho_2}\right].$$

Since all the changes are isentropic and

$$\frac{P_1}{\rho_1^{\gamma}} = \frac{P_2}{\rho_2^{\gamma}},$$

this becomes

$$\frac{\Delta w}{a_1} = \frac{2}{\gamma - 1}\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{2\gamma}}\right]$$

or

$$\frac{\Delta w}{a_2} = \frac{2}{\gamma - 1}\left[\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{2\gamma}} - 1\right].$$

In ordinary channel flow w/a_2 would give

$$\frac{w}{a_2} = \frac{2}{\gamma - 1}\left[\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right].$$

VELOCITY DISTRIBUTION IN EXPANSION WAVE WHEN PISTON STARTS FROM REST SUDDENLY

Sir Geoffrey Taylor in his paper (*Proceedings of the Royal Society*, 6th Jan. 1950, Vol. 200), has shown that the velocity distribution in an expansion wave where the piston starts from rest suddenly is linear.

A particular condition of the gas is propagated upstream with the local speed of sound, relative to the gas stream. If the gas has a velocity u in the direction of the piston, then the point where this velocity exists will move upstream in a direction opposite to the motion of the piston with a velocity $a-u$.

Since the wave is moving it is necessary to relate all velocities to a point which is moving. The most convenient point is the beginning of the expansion which moves upstream with a velocity a_1. Other points in the wave which have a velocity u move upstream with a speed $a-u$. Assuming the wave is moving into a stationary fluid and the piston is suddenly given a velocity u_2, at a time t after the piston has started, and measuring from the point where it started, the front of the wave has travelled a distance $a_1 t$ and points with a velocity u have travelled a distance $(a-u)t$. The distance between the head of the wave and points having a velocity u is given by

$$x = a_1 t - (a-u)t .$$

Dividing by $a_1 t$ to make the equation non-dimensional

$$\frac{x}{a_1 t} = 1 - \frac{a-u}{a_1}.$$

Now the velocity in the expansion wave is calculated from

$$a_1 - a_2 = \frac{\gamma - 1}{2}(u_2 - u_1) .$$

In this case $u_1 = 0$ since the wave is moving into a stationary fluid and the equation may be written

$$a_1 - a = \frac{\gamma - 1}{2} u$$

or

$$1 - \frac{a-u}{a_1} = \frac{\gamma+1}{2}\frac{u}{a_1}.$$

Therefore

$$\frac{x}{a_1 t} = \frac{\gamma+1}{2}\frac{u}{a_1}.$$

Values of u between zero and u_2 will give a linear velocity distribution in the expansion wave.

REFERENCE

1. DAWSON, L. G. (1953). Flow Changes in Gases in which Mass and Impulse are Conserved. *The Aeronautical Quarterly*, February 1953.

Note added by author in proof: ΔT is the temperature rise which would occur if the combustible material were burned under constant pressure conditions. The heat released due to combustion in any of the wave processes discussed is given by $K_p \Delta T$. With stationary combustion ΔT is equal to the change in total temperature. For detonation waves this is not true, as can be seen from equation (7). In the paper it is unintentionally implied that ΔT might represent the change in total temperature across the detonation waves, although the analysis shows that this is not the case.

An Experimental Study of a Symmetrical Aerofoil with a Rear Suction Slot and a Retractable Flap

by

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SUMMARY :

Experiments were made on an aerofoil of 25 per cent. thickness with a suction slot at the rear and a flap that could be withdrawn into the slot. The main object of the work was to investigate the effect of withdrawing the flap when *all* the air in the boundary layer was sucked into the slot. In particular, as a test of the type of anti-gust device proposed by Thwaites[1], it was required to observe the change of lift that occurred during withdrawal of the flap and a subsequent change of incidence (with the flap still withdrawn).

The flow was investigated by the use of smoke and tufts, by pressure plotting on the surface of the aerofoil, and by measurements of total and static pressures in the wake. The effects of varying incidence, flap position, and suction velocity were investigated.

It was found that the provision of a flap downstream of the enclosing streamline, so that the rear stagnation point was not in the free fluid, was all important in relation to the behaviour of an aerofoil which had a suction slot at its rounded rear.

With the flap extended *beyond* the enclosing streamline and with the ratio of suction-slot velocity to free stream velocity higher than the theoretical value, a stable flow was obtained within the theoretical incidence range. A favourable pressure gradient was observed in the slot entry and no measureable wake existed downstream of the flap. Smoke filaments, emitted from a tube projecting from the slot slightly beyond the enclosing streamline, travelled far downstream with apparently little diffusion.

When the flap was placed so that its trailing edge was *inside* the enclosing streamline, *i.e.* so that there was in theory a stagnation point in the free fluid, an instability of a three-dimensional character occurred, with a wide wake and a large drag coefficient. The lift curves were unusual and it was found that movement of the flap caused a large change of lift coefficient. There was no evidence that the lift could be maintained constant, independent of incidence, when the flap was withdrawn into the slot.

1. Introduction

1.1. PROPOSAL FOR GUST ALLEVIATION

The main purpose of this investigation was to test an interesting idea for gust alleviation which had been suggested by Thwaites[1, 2]. Thwaites proposed that a porous circular cylinder, with sufficient suction to prevent the formation of a wake behind it, could have its circulation controlled by a short, thin flap (a Thwaites flap) set normal to the surface at the rear and capable of movement around the cylinder. The Thwaites flap could also be used with an aerofoil having a rounded rear part and distributed suction over either that part or the whole of the surface. Thwaites suggested that by the use of this flap the circulation could be adjusted independently of the wing incidence and, if all the air in the boundary layer could be sucked away, the flap would generate very closely the Joukowski value of the circulation. Then, if the flap were withdrawn, the circulation, and hence the lift, might be expected to remain constant, independent of incidence changes.

Received 2nd April 1953.

With the porous circular cylinder, a further interesting flow pattern can presumably be formed. Assuming that the flap has generated a circulation and has then been withdrawn, the wind speed can be reduced to zero and there should remain a purely circulatory flow together with the "sink" flow into the surface due to the suction. That such a flow is theoretically possible was shown by Preston[3], who found exact solutions of the equations of motion for steady circulatory flow about a porous circular cylinder with suction greater than a critical value, with the cylinder either fixed or rotating. Preston[4] has also shown how such flows can be set up by rotation of the cylinder and application of suction, and he has verified theoretically that changes of suction or rate of rotation produce no change of circulation about the cylinder when the circulation has been induced initially by rotation. This work suggests that, if a Thwaites flap can be withdrawn without loss of circulation, it may be possible to produce these flows by using a Thwaites flap which is withdrawn after the circulation has been developed.

Although Thwaites's suggestion for gust alleviation is based on distributed boundary-layer suction, there

appears to be no reason why the same properties could not be achieved with a multi-slot aerofoil or one with a single rear slot. The retractable flap would be placed in the rear slot and would be capable of chordwise movement only, as distinct from the movement around the surface of the aerofoil in Thwaites's original suggestion. With this in mind, the present investigation was made on an aerofoil with a single suction-slot at the rear.

1.2. ADDITIONAL APPLICATIONS

Another matter of interest which could be investigated with this aerofoil was the question whether an aerofoil could be constructed to have no wake downstream. A smoke tube held in the model, with its outlet slightly downstream of the enclosing streamline which is obtained with this arrangement, would then emit smoke filaments that would diffuse only very slowly. This would be an excellent form of smoke emitter for visualisation of air flow. The tests were also of interest in providing information on the behaviour of a limiting form of a Griffith aerofoil with twin suction-slots, the twin slots being placed very close to the trailing edge.

1.3. OUTLINE OF THE RESEARCH

The aerofoil was developed theoretically by M. B. Glauert of the National Physical Laboratory and, because of its unusual shape, became known as a "Lobster Pot" aerofoil.

The model consisted of an aerofoil of 10 in. chord and 25 per cent. thickness with a suction slot and retractable flap at the rear. The tests were made in the 20 in. $\times$ 7½ in. wind tunnel at Cambridge University, at speeds up to 150 ft./sec. (Reynolds numbers up to 8×10^5).

The experiments described here include pressure plotting of the whole surface, smoke and tuft tests, and wake traverses giving drag coefficients by integration. Lift coefficients were obtained by integration of the measured pressure distributions, and a strain-gauge balance was also used to measure lift and pitching moment.

The effects of varying the following three parameters were investigated:—

(*i*) Incidence of the aerofoil.
(*ii*) Flap position.
(*iii*) Ratio of suction-slot velocity to free stream velocity.

Additional tests were made with several different types of flap, and also with extension strips attached to the rear of the lower surface of the aerofoil.

Notation

ζ plane of circle with sink at rear and flap on the axis
ξ horizontal axis of circle, with origin at the centre
a radius of circle of the ζ-plane
U_0 free stream velocity
u experimental velocity over aerofoil surface
Z plane of suction-slot aerofoil
x horizontal axis of aerofoil, with origin at leading edge
y vertical axis of aerofoil, with origin at leading edge
q_m theoretical velocity at a point on the aerofoil
m strength of sink on circle at $\xi = a$
U_s velocity in rear suction-slot
β design incidence of aerofoil
c chord of aerofoil, $4a > c > 3a$
X distance of trailing edge of flap from rear of aerofoil
K circulation around aerofoil
a' radius of the circle with flap as part of the periphery, $a' > a$. (Plane 2)
θ_1 polar angle of sinks in circle of radius a'. (Plane 2)
α angle of incidence of aerofoil measured on turntable
q_0 dynamic pressure of free stream $= \frac{1}{2}\rho U_0^2$
ρ density of air
H_0 free stream total pressure
P_0 static pressure in undisturbed stream
$C_p, \bar{C}_p$ pressure coefficients in rear suction-slot
L lift on the aerofoil
ΔC_L increase in lift coefficient
Q suction quantity per unit span of aerofoil
C_Q suction-quantity coefficient $= Q/(U_0 c)$
C_{Ds} sink-drag coefficient $= 2C_Q$
H_w total pressure in wake of aerofoil
P_w static pressure in wake of aerofoil
$g = (H_w - P_0)/(\frac{1}{2}\rho U_0^2)$
$p = (P_w - P_0)/(\frac{1}{2}\rho U_0^2)$
C_{Dw} wake-drag coefficient based on aerofoil chord,

$$C_{Dw} = 2\int (1-g^{\frac{1}{2}})(g-p)^{\frac{1}{2}}\, d\left(\frac{y}{c}\right)$$

$C_{Mc/4}$ Pitching moment coefficient about quarter chord point

2. Theoretical Considerations

2.1. CHOICE OF AEROFOIL

If boundary-layer suction is applied to a porous aerofoil with a Thwaites flap then, according to Thwaites[1], the formation of a wake can be prevented and the flap will generate a circulation nearly equal to the Joukowski value, *i.e.* such that the flow leaves the flap smoothly. Complete withdrawal of the flap into the aerofoil, it is suggested, will leave the circulation unchanged as long as no wake forms, even if the incidence of the aerofoil is changed. It is the main purpose of this investigation to establish whether this will prove to be the case experimentally, as previously discussed in Section 1.

Withdrawal of a flap into a porous surface is difficult, but almost any aerofoil with a rear suction-slot and a retractable flap in it could be used for a test of Thwaites's suggestion for gust alleviation. However, the suction necessary to prevent the formation of a wake may well be so large as to make the scheme uneconomical.

M. B. Glauert had developed his "Lobster Pot" aerofoil as a "disturbance-free" section, for the

production of well defined smoke filaments at high air speeds, by designing the aerofoil to give a favourable pressure gradient over the whole surface. Dr. Preston considered that the "Lobster Pot" aerofoil would make a convenient section for the present investigation, by the addition of a retractable flap along its chord line. Because this aerofoil had the thinnest possible boundary layer over its surface, it should be easy to prevent the formation of a wake behind it. Fig. 1 shows an outline of the arrangement.

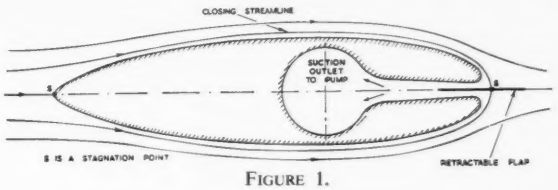


FIGURE 1.

2.2. DESIGN OF AEROFOIL

The "Lobster Pot" aerofoil is designed to have no unfavourable pressure gradient on any part of the surface, for a certain limited range of incidence. This is achieved by the "sink" effect of the slot, where the potential-flow velocity rises to a maximum and the pressure falls to a minimum. In general, the required suction flow will exceed the flow in the boundary layer. If the surface is free from waves and protruberances it should be possible to maintain laminar flow within the designed incidence range, and hence keep the boundary layer very thin. Outside the designed incidence range, transition will occur, and much greater suction quantities will be required to remove the thicker turbulent boundary layer.

The Lighthill[5] method of two-dimensional aerofoil design was used by M. B. Glauert to transform a circle having a sink and flap at its rear into the "Lobster Pot" aerofoil. The profile of the 25 per cent. thick aerofoil is shown in Fig. 2 and its co-ordinates are given in Table I. The aerofoil was designed so that at zero incidence the velocity increased steadily from the leading edge to a point inside the slot. The low-drag range, for which laminar flow should be possible over the whole surface, is $-4° < \alpha < +4°$, $(-0{\cdot}548 < C_L < +0{\cdot}548)$. The theoretical velocity in the slot was 1·345 × the free stream velocity, giving a C_Q of 0·0337 and a critical Mach number of 0·622. It was found that a 10 in. chord model would enable the internal ducting and retractable flap to be incorporated in it, and the necessary suction supply could be handled by equipment in the laboratory.

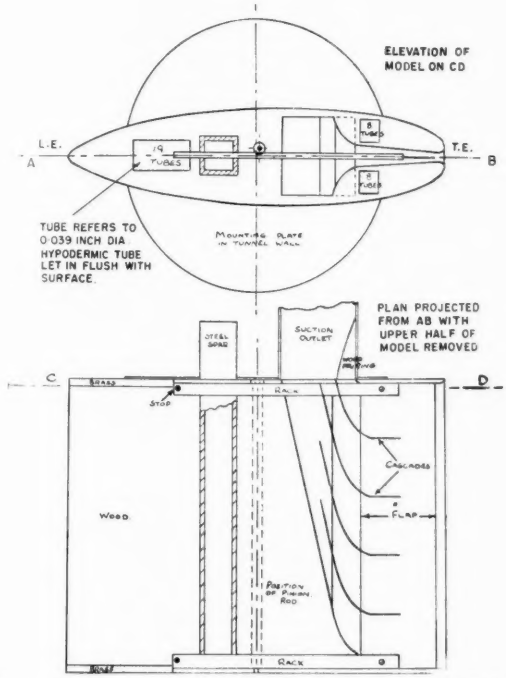


FIGURE 2. Drawing of two-dimensional model of "Lobster Pot" aerofoil.

The details of thinner "Lobster Pot" aerofoils were thought to be of interest and calculations were made for a 13¼ per cent. thick section. The theoretical slot velocity and slot width both decrease with thickness/chord ratio and the 13¼ per cent. section has a C_Q of 0·009 while the limit of the low-drag range is $\alpha = 1\frac{3}{4}°$ $(C_L = 0{\cdot}209)$. The corresponding theoretical slot velocity is 1·1723 × the free stream velocity and the critical Mach number is 0·743. The co-ordinates of the 13¼ per cent. thick section are given in Table II.

2.3. FURTHER CALCULATIONS

(a) Position of Stagnation Point at Rear of Aerofoil. An important feature of the flow about this aerofoil is the enclosing streamline which surrounds the rear of it

TABLE I

CO-ORDINATES OF "LOBSTER POT" AEROFOILS

25 PER CENT. THICK "LOBSTER POT" SECTION

x/c	y/c
0	0
0·00216	0·00513
0·00432	0·0081
0·0108	0·0146
0·0314	0·0284
0·0611	0·0427
0·1457	0·0714
0·2575	0·0960
0·321	0·1059
0·457	0·1198
0·527	0·1233
0·597	0·1243
0·664	0·1228
0·729	0·1190
0·844	0·1038
0·892	0·0930
0·933	0·0803
0·987	0·0505
0·998	0·0341
1·000	0·0297
1·000	0·0254
0·998	0·0211
0·988	0·0146
$-\infty$	0·0125

TABLE II

13¼ PER CENT. THICK "LOBSTER POT" SECTION

x/c	y/c
0	0
0·00178	0·00261
0·00374	0·00416
0·00957	0·00708
0·02851	0·01454
0·05667	0·02242
0·1380	0·03793
0·1895	0·04499
0·2470	0·05131
0·3755	0·06099
0·4439	0·06408
0·5482	0·06627
0·5828	0·06632
0·6505	0·06538
0·7761	0·05942
0·8316	0·05451
0·9225	0·04130
0·9562	0·03328
0·9960	0·01528
1·000	0·00811
1·000	0·00751
0·9992	0·00571
0·9977	0·00441
$-\infty$	0·00371

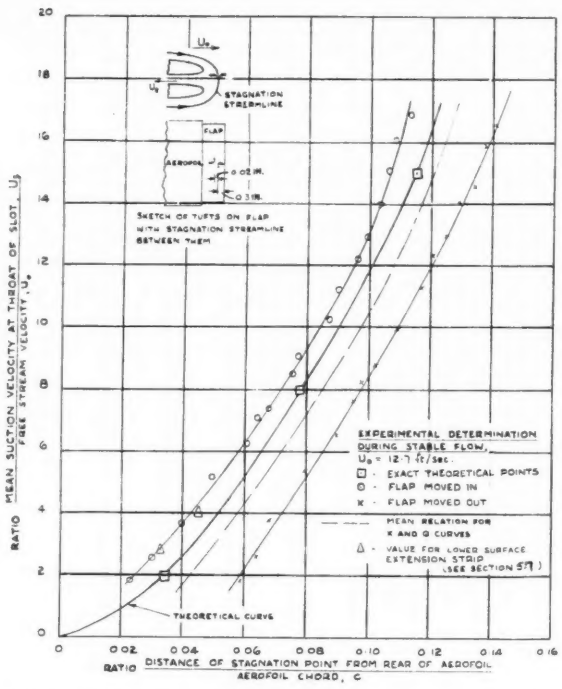

STAGNATION STREAMLINE
FLAP
AEROFOIL
0·02in.
0·31in.
SKETCH OF TUFTS ON FLAP WITH STAGNATION STREAMLINE BETWEEN THEM
EXPERIMENTAL DETERMINATION DURING STABLE FLOW,
EXACT THEORETICAL POINTS
FLAP MOVED IN
FLAP MOVED OUT
MEAN RELATION FOR × AND ⊙ CURVES
VALUE FOR LOWER SURFACE EXTENSION STRIP (SEE SECTION 5?)
THEORETICAL CURVE
RATIO MEAN SUCTION VELOCITY AT THROAT OF SLOT, U_s / FREE STREAM VELOCITY, U_0
RATIO DISTANCE OF STAGNATION POINT FROM REAR OF AEROFOIL / AEROFOIL CHORD, c
0 0·02 0·04 0·06 0·08 0·10 0·12 0·14 0·16
0 2 4 6 8 10 12 14 16 18 20

FIGURE 3. Relation between position of rear stagnation point and suction velocity ratio U_s/U_0.

and separates the air that is sucked into the slot from the air flowing downstream. Thus there will be a stagnation point in the fluid where this enclosing streamline crosses the axis. The enclosing streamline is of importance in the development of a "disturbance free" smoke emitter since the smoke outlet must be situated at a point slightly downstream of the enclosing streamline, if the smoke is to be carried downstream and not into the slot. If the flap extends sufficiently far from the aerofoil the stagnation point occurs on the flap, but when the flap is withdrawn the stagnation point would be expected to remain in the free fluid, at a fixed distance behind the aerofoil.

The distance of the rear stagnation point from the aerofoil is not obtained during the design of the section by the Lighthill method[5]. Further calculations were therefore made to determine this distance.

In these calculations it is necessary to consider both the plane of the circle with sink at its rear, from which the aerofoil was transformed, and the plane of the aerofoil. The axial velocity in the plane of the circle can easily be found, but to determine the axial velocity at the rear of the aerofoil it is necessary to replace the surface of the aerofoil by a vortex sheet with strength equal to the surface velocity, and to carry out integrations over the whole surface for different points on the aerofoil axis. Complete correspondence between the two planes can be obtained by starting from the corresponding stagnation points. Use of the velocity potential thus enables other corresponding points to be determined. The ratios of the velocities at corresponding points then give the modulus of the transformation.

The position of the stagnation point on the rear axis of the aerofoil depends on the suction quantity, the effect of increased suction being to move the stagnation point farther away from the aerofoil. Thus, at increased suctions, it is necessary to determine the position of the stagnation point for the circle plane at the new suction value and then, using the relation between corresponding points in the two planes, to find the distance of the corresponding stagnation point from the rear of the aerofoil.

Calculations made in this way showed that the distance of the stagnation point from the rear of the aerofoil at the theoretical suction flow ($U_s/U_0 = 1{\cdot}345$) was $0{\cdot}0266c$. At higher suctions giving $U_s/U_0 = 2{\cdot}0$, $8{\cdot}0$ and $15{\cdot}0$, this distance became $0{\cdot}0339c$, $0{\cdot}0777c$ and $0{\cdot}1145c$ respectively. These values are plotted in Fig. 3. The value $U_s/U_0 = 2{\cdot}0$ was used in most of the experimental work, and the distance of the stagnation point from the rear of the aerofoil at this suction velocity was found to be $0{\cdot}0348c$ when the thickness of the flap in the slot was taken into account.

(*b*) *Effect of Unequal Suction Flows on the Two Sides of the Flap.* On this aerofoil the retractable flap divides the rear suction slot into two halves, and virtually produces upper and lower sinks at the trailing edge of the aerofoil. Ideally, these sinks should be of equal strength, but due to errors in manufacture and the fact that the upper and lower boundary layers are not of equal thickness, the flows on the two sides of the flap may not be the same. Accordingly, the lift actually obtained on the aerofoil may differ from the value which should have been obtained assuming equal sink flows at the slots. In fact, boundary-layer suction at the rear of an aerofoil is an effective method[6, 7, 8] of changing lift by controlling the position of the rear stagnation point.

To obtain an idea of the difference in lift that is possible with unequal suction flows, the problem was investigated theoretically. For this it is necessary to separate the two effective sinks in the plane of the aerofoil and further transformations, shown in Fig. 4, make this possible. The original plane of the circle is transformed into a flat plate (Plane 1); the flap along the rear axis of the circle is then still part of the axis of the plate in the transformed plane. With a suitable change of origin, namely to the mid-point of the extended plate, another circle can be formed (Plane 2) having a greater radius than the original one. The sinks in this circle are separated and situated on the periphery at equal small angular displacements from the trailing edge generator.

TABLE III

Ratio of distance of flap from rear of aerofoil to chord, X/c	*Ratio of radii a′/a*	*Angle θ_1 from OD in Plane 2 (Fig. 4)*
0·025	1·0041	7° 16′
0·034	1·0065	9° 14′
0·050	1·0123	12° 38′
0·100	1·0369	21° 45′
0·150	1·0671	29° 2′

The enclosing streamline meets the surface of the circle of Plane 2 at stagnation points C_1 and C_2, which are situated at polar angles $\pm\gamma$ from OD.

Hence it will be seen that as the flap extends farther downstream from the rear of the aerofoil, the two rear sinks of Plane 2 move farther apart. The ratio of the radii of the circles a'/a and the polar angle of the sinks θ_1, are given in Table 3, for the five flap positions used in the experiments.

The theory shows that, with unequal suction flows at the sinks of Plane 2, change of flap position alters the lift and the symmetrical aerofoil does not have zero lift at $\alpha = 0°$. In fact, the increase in lift coefficient on the aerofoil is:—

$$\Delta C_L = 0{\cdot}020 \frac{\bar{C}_p}{\bar{U}_s} \times \cot\frac{\theta_1}{2} \quad . \quad . \quad (1)$$

where $\bar{C}_p$ = a pressure coefficient in the rear suction slot relating the differing suction flows above and below the flap. *See* Section 4.3(*d*). $\bar{U}_s = U_s/U_0$ = suction velocity ratio in the rear slot.

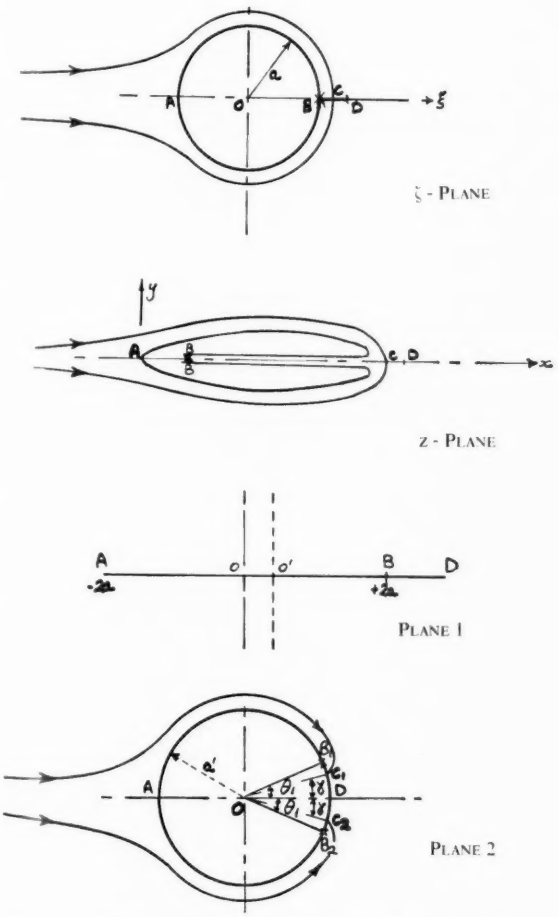


FIGURE 4. Further transformations from the plane of the original circle.

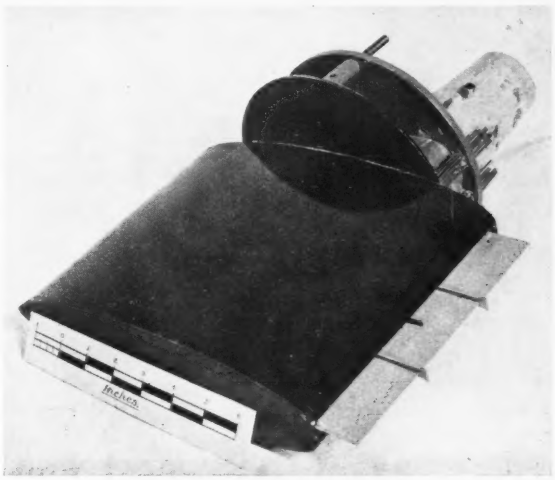


FIGURE 5.

3. Experimental Equipment

3.1. DESCRIPTION OF MODEL AND TEST ARRANGEMENT

The two-dimensional model (Fig. 2) was made by fly-cutting a wood block with a cutter of the requisite profile. The wing was made in two halves, joined together by two brass end plates. A steel spar to carry the forces on the model was fastened to the outer end plate and carried through the model to the mounting disc in the wind-tunnel wall. As shown in Fig. 2, the air sucked through the slot entered a diffuser of 8° included angle, followed by a constant-area duct of the type developed by Rawcliffe[9], with four sheet-brass guide-vanes to turn the flow. The suction duct from the wing was led to a $\frac{3}{4}$-radius flow meter[10], then to a bleed valve for controlling the suction flow, and finally to the input side of a Marshall blower.

The first axially retractable flap was made from a piece of steel, 0·012 in. thick and 2 in. wide. The flap had a sharp trailing edge and was brazed to two lengths of rack which ran in wood grooves in the wing and in slots in the brass end plates. A length of pinion rod engaged the rack of the flap, and the trailing edge of the flap could be set at any distance up to 1·6 in. downstream of the rear of the aerofoil. The aerofoil on the mounting disc is shown in Fig. 5.

The aerofoil carried a large number of small hypodermic tubes let in flush with the surface. These were placed on both surfaces symmetrically with respect to the leading edge so that the load distribution could be obtained by taking differences of the respective manometer readings. A smoke outlet was provided flush with the upper surface at 0·05c from the leading edge. Three other smoke tubes were placed across the span in the rear slot, with their outlets slightly downstream of the enclosing streamline at $U_s/U_0 = 2{\cdot}0$. A paraffin smoke generator was fixed on the side of the tunnel to supply smoke to these outlets.

A wake-traverse gear was supported from the tunnel roof. This could be used for traversing either a pitot or a static tube in a plane $11\frac{3}{4}$ in. from the rear of the

FIGURE 6.

aerofoil. A pitot tube was also placed upstream of the model for comparison with the wake tube.

3.2. MODIFICATIONS OF THE MODEL

(*a*) *Different Types of Flaps.* Most of the experimental work was done with the flap described in the previous section. Other steel flaps were as follows:— (Numbers 1 to 4 had sharp trailing edges)

(1) A perforated flap of 2 in. chord and 0·020 in. thick. About 200 holes were drilled in this at $\frac{1}{4}$ in. centres along the span and chord (Fig. 6).

(2) A short flap of $\frac{1}{4}$ in. chord and 0·022 in. thick (Fig. 6).

(3) A flap of triangular cross-section of $\frac{1}{4}$ in. chord. Base of triangle $=\frac{1}{8}$ in. (apex as trailing edge).

(4) A 2 in. chord flap with a central V-notch cut out of its trailing edge and thickness = 0·022 in. Base of notch at trailing edge = 2 in. and height of notch = 1 in. (Fig. 6).

(5) A Thwaites leading edge flap[13] of chord 0·37 in. Sharp leading edge and maximum thickness of 0·12 in. at its rear, fitting against the leading edge of the model.

(6) The original plate flap with lower half of slot blanked off. Trailing edge of flap = 0·625 in. from rear of aerofoil.

(*b*) *Trailing Edge Extension Strips.* Hardwood fairings were attached to the trailing edge of the lower surface, the increases in the chord of the lower surface being 0·0192*c*, 0·0324*c* and 0·0451*c* with the three strips used.

3.3. DESCRIPTION OF THE STRAIN-GAUGE BALANCE

The model wing and its turntable were carried by two high-carbon steel strain-gauge elements, which had their centre lines situated at the leading edge and trailing edge of the aerofoil (Figs. 7 and 8). Strain gauges were attached to each of the eight rectangular faces of the elements, where the cross-section was 1 in. $\times \frac{1}{4}$ in. The 1 in. side of the element was parallel to the chord line so that the stiffness of the balance was small in the lift direction and large in the drag direction.

An air clearance was allowed between the scale plate of the turntable and the tunnel wall, to give freedom of movement at that position for the expected range of lift and drag. This necessitated a sealing box on the outside of the tunnel to prevent air leaking into the tunnel. The cover of this box was made from thick dural sheet and the outer flanges of the strain-gauge elements were rigidly bolted to it, so that the balance and model were carried by the cover plate.

Incidence and flap adjustments were made by two hand controls, accessible through small airtight doors in the sealing box. The suction connection to the model was made with a corrugated rubber tube (Fig. 7).

The balance was designed to measure a maximum lift force of 10 lb. to an accuray of one per cent., while the pitching moment accuracy was about $\frac{1}{2}$ lb. in. when the centre of pressure was anywhere within the chord.

The circuit used with the doubled strain gauges is given in Fig. 9. The balance was calibrated by dead-weight loading.

4. Results of Experiments

4.1. UNIFORMITY OF SUCTION AT REAR SLOT

The spanwise distribution of suction flow was measured by means of exploring tubes which faced downstream relative to the suction flow into the slot (Fig. 6). The results of these measurements were expressed as a pressure coefficient C_p, defined as the difference between the local static pressure and the atmospheric pressure, divided by the mean dynamic pressure at the throat of the slot. It was then assumed that the variation in $\sqrt{C_p}$ across the slot was proportional to the velocity variation.

With the tunnel stream in operation, traverses above and below the flap showed that more suction flow was entering above the flap than below it. The spanwise distribution was considered satisfactory.

4.2. FLOW VISUALISATION AND EFFECT OF DIFFERENT FLAPS

(*a*) *Development of the Flow about the "Lobster Pot" Aerofoil.* The flow was studied visually at all stages of the investigation, both by use of smoke filaments from the outlets described in 3.1, and by means of tufts of goose-down cemented on to the flap and the rear of both wing surfaces.

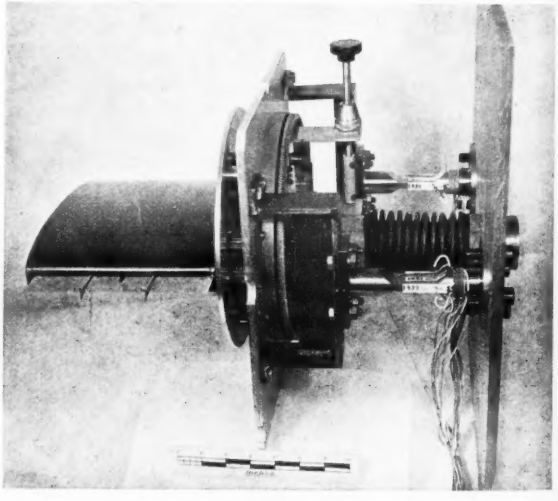


FIGURE 7.

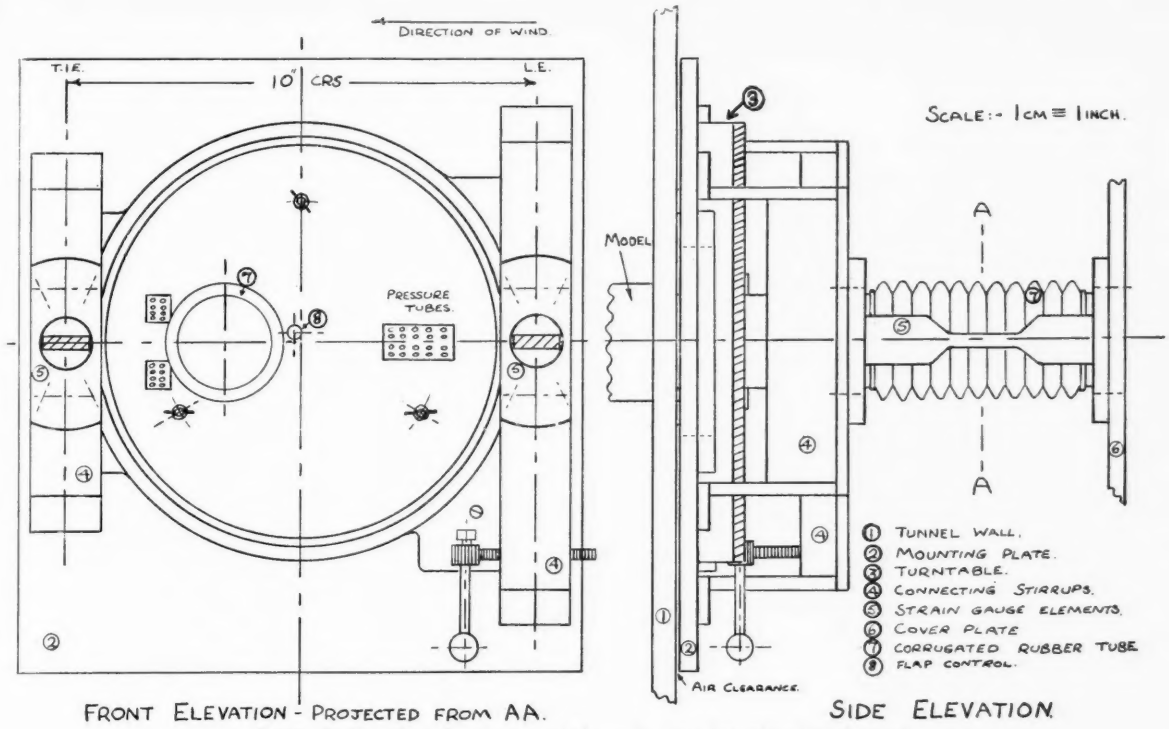


FIGURE 8. Drawing of strain-gauge balance for two-dimensional wind tunnel.

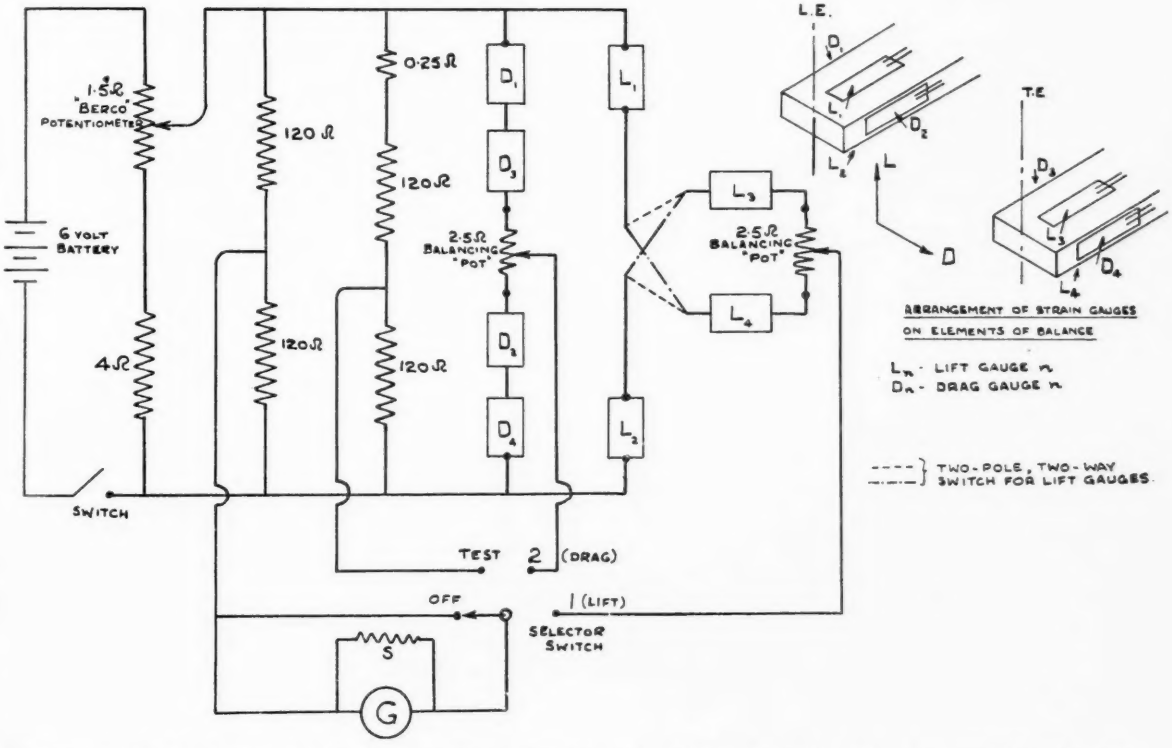


FIGURE 9. Circuit diagram of Cambridge Instrument Company strain bridge. Reduced circuit when used in conjunction with the doubled gauges of a strain-gauge balance.

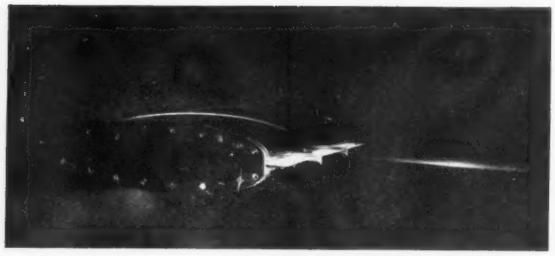


FIGURE 10. $\alpha=0°$, $X/c=0{\cdot}150$, $U_s/U_0=2{\cdot}0$.

With either the suction flow or the tunnel stream in operation separately, the flow over the surface was laminar and steady. Without suction, the flow separated from the aerofoil surface and formed a moderately thick turbulent wake. Outside this wake, the flow was steady.

With the aerofoil at zero incidence and the flap extending to $0{\cdot}160c$ (from the rear of the aerofoil) increase of suction gradually moved the separation point to the rear of the aerofoil. At $U_s/U_0=1{\cdot}81$, a favourable pressure gradient was obtained over both surfaces, the flow entering the slot smoothly.

At this section flow, however, changes of incidence within the design range of $\pm4°$ caused adverse pressure gradients sufficient to produce separation. When the suction was increased to $U_s/U_0=2{\cdot}0$, favourable pressure gradients were obtained over the whole surface of the aerofoil for an incidence range of $\pm4°$.

The aerofoil design[5], using potential flow theory, gave $U_s/U_0=1{\cdot}345$ for favourable pressure gradients over the incidence range $\pm4°$. The difference between this suction velocity and the value required in the experiments to prevent separation may have been due to the aerofoil boundary layers.

With $U_s/U_0=2{\cdot}0$ and the flap extending well downstream, smoke emitted from the three outlets fixed in the rear slot gave rise to very "clean" and sharply defined filaments behind the aerofoil. Thus, a successful "disturbance free" smoke emitter has been produced. The filaments extended for several chords behind the aerofoil, only becoming diffused by the turbulence of the stream in the diffuser of the wind tunnel.

Figure 10 shows the laminar form of a smoke filament over the upper surface of the aerofoil and one sharply defined smoke filament downstream of the aerofoil. Fig. 11 shows the flow about the aerofoil as in Fig. 10, with a streamer in the wake and tufts cemented to the aerofoil.

It was found that the transverse stagnation line was curved in plan view, being farther from the aerofoil at the ends than at mid-span. This was probably due to the presence of the tunnel-wall boundary layers making U_s/U_0 greater near the tunnel walls.

Increase of U_s/U_0 beyond 2·0 increased the incidence range for which the pressure gradients were favourable.

(*b*) *Effect of Flap Withdrawal.* With the aerofoil placed at a small positive incidence and with $U_s/U_0=2{\cdot}0$, the flap was slowly retracted. When the flap was at $0{\cdot}050c$, the smoke filaments from the three rear tubes diverged laterally and small vertical oscillations with some rotation were observed in them. This showed that trailing vorticity was present downstream of the aerofoil, corresponding to the breakdown of two-dimensional flow, and the circulation was asymmetric across the span. When the flap was retracted to $0{\cdot}034c$, the unsteadiness in the smoke filaments increased and considerable diffusion of the smoke occurred. With $U_s/U_0=2{\cdot}0$ the rear stagnation point was at the trailing edge of the flap for this flap position.

Further retraction of the flap to $0{\cdot}025c$ produced the totally unexpected result of an unstable flow of great violence. Loud beating noises were audible in the tunnel and in the suction connections and the tunnel stream and suction flow were both very unsteady. This unstable flow was of a *three-dimensional* nature, taking several *different* forms, although the complex flow patterns were difficult to observe. A cotton streamer showed that the wake nearly filled the tunnel when the flap was retracted. A photograph of the unstable flow is shown in Fig. 12.

The cause of this violent instability was the removal of the stagnation point from the trailing edge of the flap into the free fluid behind the aerofoil. Further evidence to confirm this statement is presented later. However, the reason why a stagnation point cannot exist in the free fluid in a stable manner is not known.

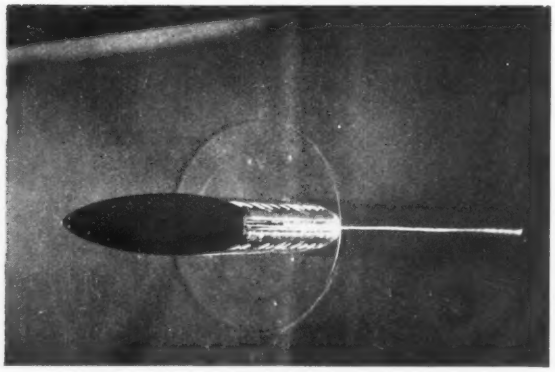


FIGURE 11. $\alpha=0°$, $X/c=0{\cdot}100$, $U_s/U_0=2{\cdot}0$.

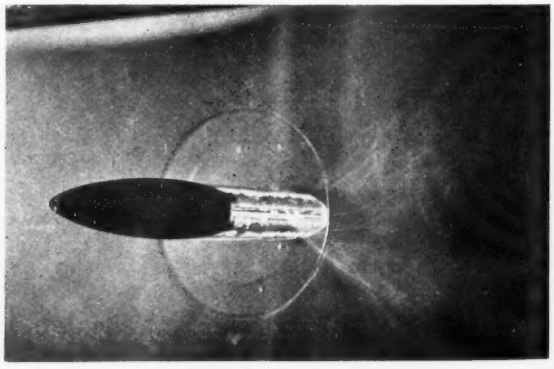


FIGURE 12. $\alpha=0°$, $X/c=0$, $U_s/U_0=2{\cdot}41$. Exposure 1 sec.

Sometimes, during the unstable flow, the upper-surface smoke filament was still of laminar form for about half the length of the chord. Over the remainder of the aerofoil the filament was bent in a spanwise direction into a region where "stable" suction took place, and if this region moved along the span with the oscillations of the flow, then the smoke filament also moved. With rapid oscillation of the general flow, the smoke filament swept to and fro across the span.

Withdrawal of the flap until it was completely inside the slot increased the unsteadiness further, and a change of incidence then produced a violently unsteady flow. Similarly, this unsteadiness of flow could be produced with the flap farther out from the aerofoil than $0{\cdot}034c$, by increasing the suction quantity and thus moving the stagnation point away from the aerofoil. In view of the very close agreement between the actual and theoretical positions of the rear stagnation point at $U_s/U_0 = 2{\cdot}0$, the actual position of the stagnation point at higher suctions was determined by experiment.

For this determination, two goose-down tufts were cemented to the flap as shown in Fig. 3, and the flap was then moved inwards until the inner tuft pointed into the slot and the outer one downstream.

The stagnation point was then at the mid-point between the tufts; hence the position of the stagnation point was found for a range of values of U_s/U_0 from 2 to 17. As the tufts showed a hysteresis effect depending on the direction of flap movement, the position of the stagnation point was found for both inward and outward movements of the flap. The mean curve for the two flap movements is included in Fig. 3 and compares favourably with the calculated values.

With U_s/U_0 less than 2·0, retraction of the flap produced an instability of different form. In this case the unstable flow was two-dimensional and the flow pattern was not unlike that for the aerofoil without suction.

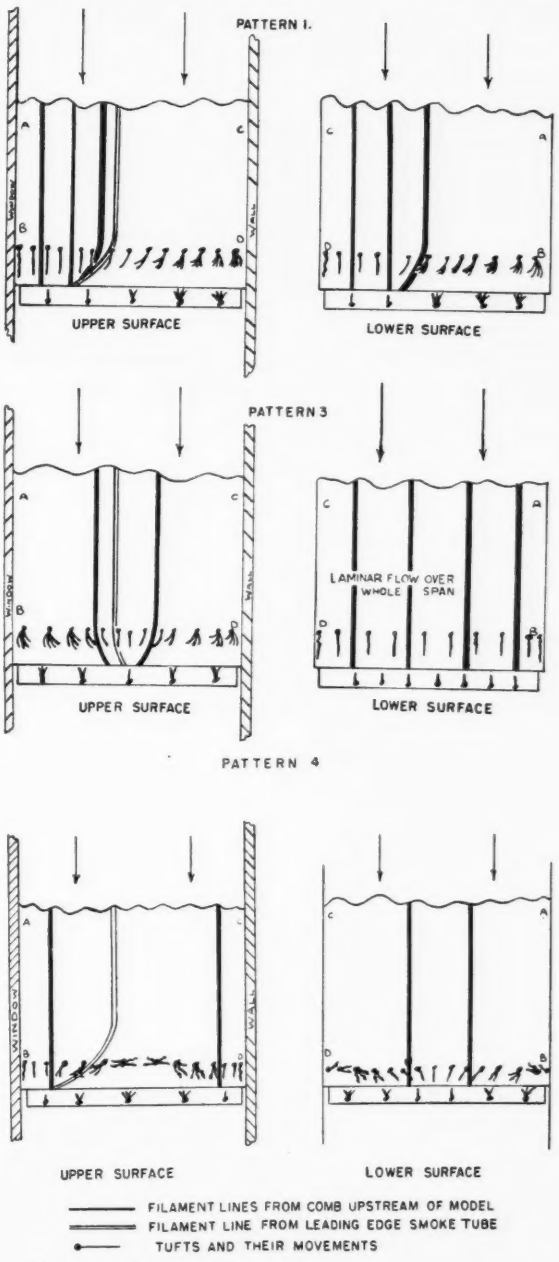
(*c*) *Unstable Flow Patterns.* As stated previously, there were several different forms of unstable flow. Altogether, eight different unstable flow patterns were observed when the flap was retracted. It is impossible to give definite details of these patterns as they were obtained both as transient states and as fixed ones.

Patterns 1, 3 and 4 are sketched in Fig. 13 in terms of smoke filaments and movement of tufts. The other five patterns, which are related to Patterns 1, 3 and 4, are explained underneath the figure. The three-dimensional nature of the instability and the large variation in circulation along the span will be apparent from these sketches.

(*d*) *Effect of Different Flaps.* The different flaps have been described in Section 3.2(*a*). When no flap was fitted in the rear slot, it was found that stable flow was impossible for any value of the suction velocity other than zero. With a suction flow the rear stagnation point was in the free fluid and instability always occurred.

When the perforated flap was fitted in the slot and was retracted so that any one of the spanwise lines of perforations corresponded to the mid-span position of the stagnation line, the flow went unstable for a very short time and then re-established itself. This re-stabilisation was probably due to the curve of the stagnation line.

With the short rectangular flap of $\frac{1}{4}$ in. chord the flow was unstable when the leading edge of the flap was in line with the rear of the model at $U_s/U_0 = 2{\cdot}0$. Winding the flap farther out stabilised the flow at first, since



Pattern 1*a* is the reverse of pattern 1.
Pattern 2 is periodic and made up of patterns 1 and 1*a*.
Pattern 3*a* is the reverse of pattern 3.
Pattern 4*a* is the reverse of pattern 4.
Pattern 5 is periodic and made up of patterns 4 and 4*a*.

FIGURE 13. Instability patterns.

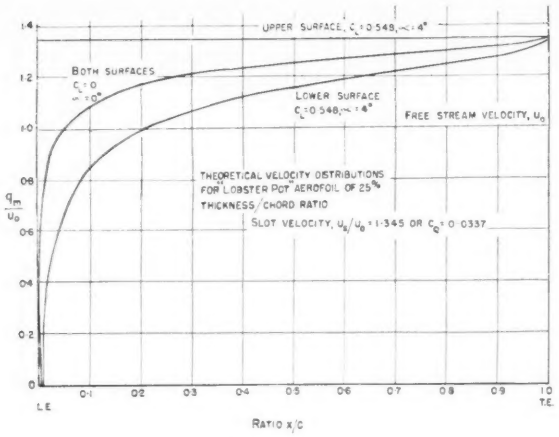


FIGURE 14.

the trailing edge of the flap was more than $0{\cdot}025c$ from the rear of the aerofoil. However, when the gap between the leading edge of the flap and rear of the model equalled $0{\cdot}025c$, an instability of reduced violence was formed and the disturbances of the tufts, smoke filaments and streamer were much less than those observed on retraction of a solid or long flap.

With the short triangular-section flap, a gap between the flap and aerofoil of $0{\cdot}041c$ could be achieved. With $U_s/U_0=2{\cdot}0$ the reduced instability was still present at this flap position, and the rear stagnation line was in the free fluid between the flap and the aerofoil. The chord of this flap was then progressively reduced at its leading edge, to determine the minimum chord for stable flow, with a gap of $0{\cdot}020c$ between the aerofoil and the flap and $U_s/U_0=2{\cdot}0$. Stability was still maintained with a flap of chord $0{\cdot}0141c$, but at a chord of $0{\cdot}0108c$, the flow could not be termed stable. This minimum chord for stability appears to be limited by (*i*) the character of the flow near a stagnation point and (*ii*) the effect of the curvature in the transverse stagnation line.

The flap with a central V-notch was used to simulate the spanwise withdrawal of a flap. At $\alpha=0°$, as the flap was retracted from its outermost position, the tufts at the centre of the span showed separation when the peak of the notch was in line with the rear of the aerofoil. The local separation on the aerofoil surfaces gradually spread towards the tunnel walls as the flap was further retracted, until at $X/c=0{\cdot}025$ the full instability ensued.

With the Thwaites leading edge flap[13] in place, the flow was stable when the rear flap projected downstream, but became very unstable when the rear flap was retracted.

When the lower half of the suction slot was blocked by a strip along the span of the flap, the flow with suction was found to be stable. The tufts on the upper surface showed an inflow and those on the lower surface an outflow, indicating that trailing vorticity was present behind the aerofoil. Retraction of this flap into the slot produced an instability of less violence than that found when the lower slot was also operative. Thus the asymmetric suction arrangement apparently had some restrictive effect on the oscillations of the free stagnation point.

4.3. PRESSURE DISTRIBUTIONS

(*a*) *Theoretical Distributions.* The velocity distributions for the aerofoil at $\alpha=0°$ and 4° were given by M. B. Glauert and are reproduced here as Fig. 14. In theory, the velocity increases over the whole surface into the slot at $\alpha=0°$, while at the limit of the design range ($\alpha=4°$) there is a nearly constant velocity over the upper surface.

In Fig. 15, the theoretical velocity distribution at $U_s/U_0=2{\cdot}0$ and $\alpha=0°$ is compared with the experimental points for both surfaces when the flap was at $0{\cdot}150c$. A favourable pressure gradient existed over the aerofoil, and the comparison between theory and experiment was good except near the rear. The velocity of the air near the wall of the slot was higher than the mean slot velocity, probably because of the sharp bends into the slot and the reduction of slot width due to the boundary layers on the walls.

(*b*) *Experimental Distributions.* Pressure distributions were recorded at incidences from $-6°$ to 6° at a tunnel speed of 120 ft./sec. At flap positions between the fully extended position and the critical position for instability, there was a small change in the magnitude and distribution of the pressures, suggesting a change of lift with flap movement. At $X/c=0{\cdot}025$, the flow was

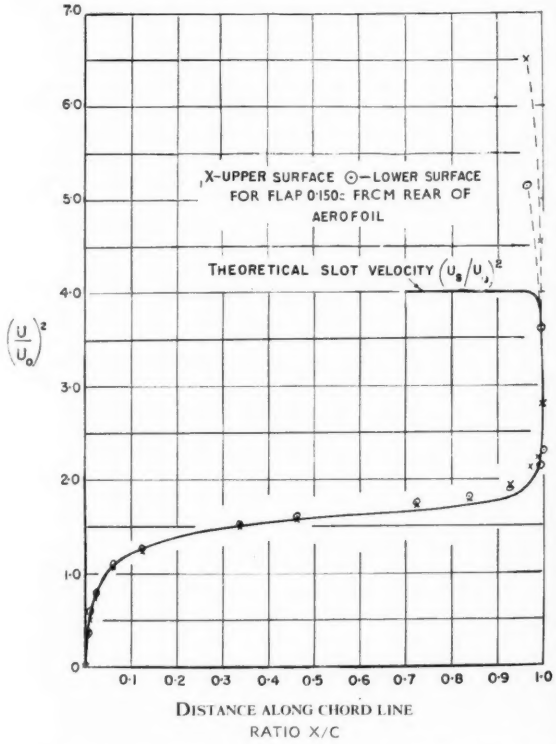


FIGURE 15. Theoretical and experimental velocity distributions for $\alpha=0°$ and $U_s/U_0=2{\cdot}0$.
Theory ——— Experiment ⊙X.

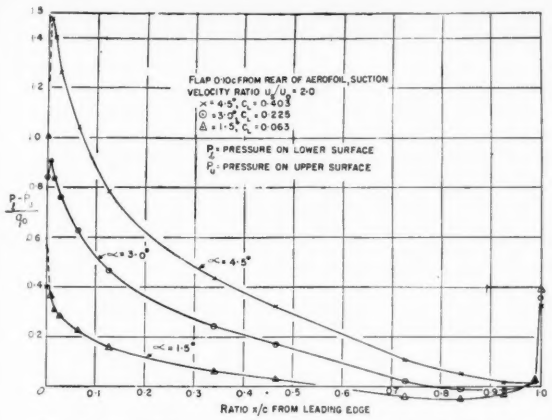

FLAP 0·10c FROM REAR OF AEROFOIL, SUCTION VELOCITY RATIO U_s/U_0 = 2·0
α = 4·5°, C_L = 0·403
α = 3·0°, C_L = 0·225
α = 1·5°, C_L = 0·063
P_L = PRESSURE ON LOWER SURFACE
P_U = PRESSURE ON UPPER SURFACE
$\frac{P-P_L}{q_0}$
α = 3·0°
α = 4·5°
α = 1·5°
RATIO x/c FROM LEADING EDGE

FIGURE 16. Effect of incidence on load distribution.

so unsteady that the manometer readings were very uncertain.

The effect of U_s/U_0 on the pressure distribution was also investigated. This was done for stable flow, with the flap at 0·100c, for values of U_s/U_0 from 0 to 4·5.

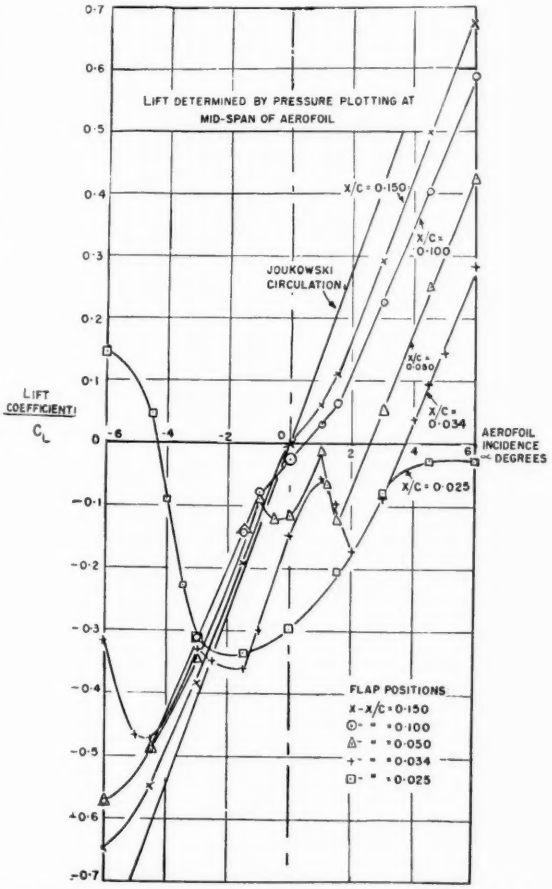


FIGURE 18. Lift curves for five flap positions when suction velocity ratio $U_s/U_0 = 2·0$.

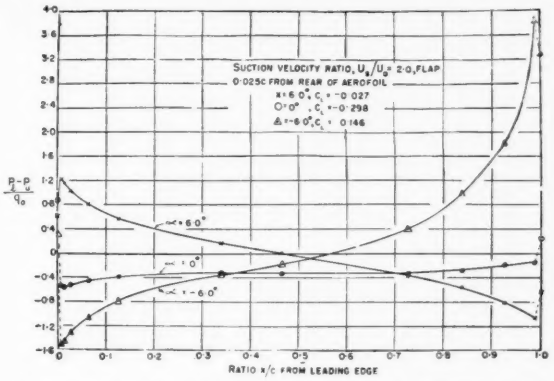

SUCTION VELOCITY RATIO, U_s/U_0 = 2·0, FLAP 0·025C FROM REAR OF AEROFOIL
α = 6·0°, C_L = 0·027
α = 0° , C_L = −0·298
α = −6·0°, C_L = 0·146
$\frac{P-P_L}{q_0}$
α = 6·0°
α = 0°
α = −6·0°
RATIO x/c FROM LEADING EDGE

FIGURE 17. Effect of flap position on load distribution at different incidences.

(*c*) *Load Distributions.* In determining the load distributions, no account was taken of the difference in pressure between the walls of the rear slot.

Figure 16 shows the effect of aerofoil incidence on the load distribution for stable flow, with the flap at $X/c = 0·100$ and $U_s/U_0 = 2·0$. When the unstable flow was developed by retracting the flap the load distributions were as shown in Fig. 17.

(*d*) *Pressure Distributions in the Rear Suction Slot.* Pressure measurements were made at two positions in the rear suction slot. The pressure coefficient $\bar{C}_p$ was defined as the difference in pressure between the lower and upper walls of the slot, divided by the dynamic pressure of the stream in the slot.

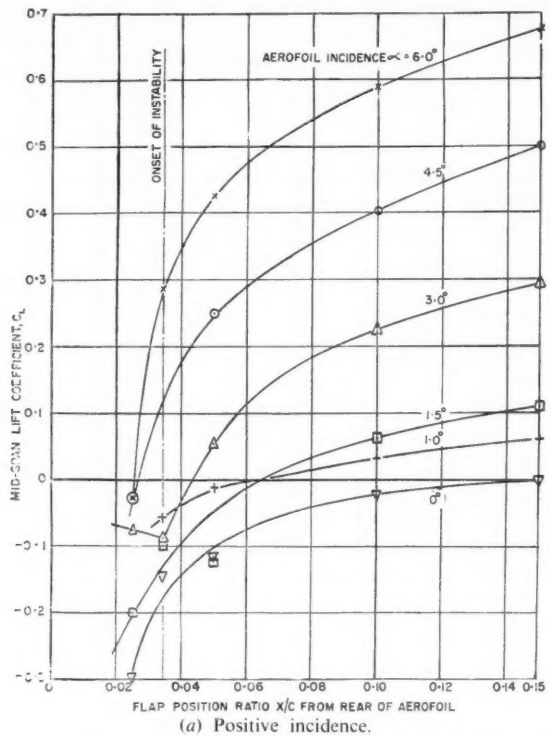
Ideally, the velocities outside the boundary layers on the upper and lower surfaces of the slot should be equal, but in practice this was not found. It was found that the pressure coefficient varied markedly with the three test variables, namely aerofoil incidence, flap position and suction velocity ratio.

4.4. LIFT DERIVED FROM PRESSURE DISTRIBUTION

Lift coefficients were determined by integration of the load distributions, for the section of the aerofoil at mid-span. Although in the stable condition the flow was nearly two-dimensional, it should be remembered that when instability occurred the flow was of a three-dimensional nature.

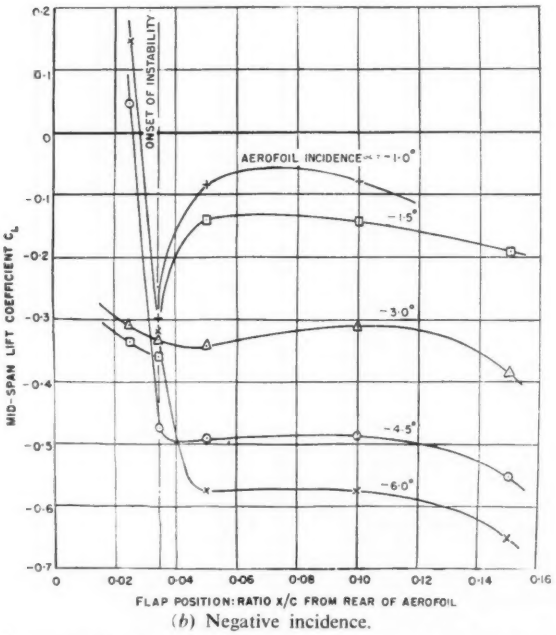
(*a*) *Results for Stable and Unstable Flows at* $U_s/U_0 = 2·0$. Fig. 18 shows lift curves for $X/c = 0·150$ to 0·025 over an incidence range from $-6°$ to $+6°$, for $U_s/U_0 = 2·0$. The theoretical lift curve for Joukowski circulation is also included. It will be seen that for any given incidence the lift did not remain constant as the flap was withdrawn, and that a large negative lift coefficient was found at zero incidence for the three inner flap positions. In all cases, the lift coefficient was less than that corresponding to the Joukowski circulation.

During stable flow, the change of lift with flap movement occurs because of the differing sink strengths above and below the flap and because of the onset of trailing vorticity at $X/c = 0·050$. The large change of



(*a*) Positive incidence.

FIGURE 19 (*a*). Relations between lift coefficient and flap position for zero and positive incidence values. Lift coefficient determined by pressure plotting at mid-span of the aerofoil. Suction velocity ratio $U_s/U_0 = 2{\cdot}0$.



(*b*) Negative incidence.

FIGURE 19 (*b*). Relations between lift coefficient and flap position for negative incidence values. Lift coefficient determined by pressure plotting at mid-span of the aerofoil. Suction velocity ratio $U_s/U_0 = 2{\cdot}0$.

lift with unstable flow shows that Thwaites's[1] suggestion for gust alleviation, by withdrawal of a flap into a suction slot after the circulation has been generated by the edge of the flap, does *not* work in practice because vorticity escapes via the wake caused by the free and unstable rear stagnation point.

Figures 19(*a*) and 19(*b*) show the variation of C_L with X/c, at $U_s/U_0 = 2{\cdot}0$. Retraction of the flap from $X/c = 0{\cdot}150$ to $0{\cdot}025$ produced changes of C_L of 0·7 at $\alpha = 6{\cdot}0°$ and 0·8 at $\alpha = -6{\cdot}0°$.

(*b*) *Effect of Suction Velocity.* For this test the flap was placed at $X/c = 0{\cdot}100$, so that stable flow would be maintained throughout. Values of U_s/U_0 from 0 to 2·0 were obtained at $U_0 = 120$ ft./sec., while values of U_s/U_0 from 2·5 to 4·5 were investigated at $U_0 = 53{\cdot}8$ ft./sec. The relationships between mid-span lift coefficient and U_s/U_0 are given in Figs 20(*a*) and 20(*b*), for incidences between $-6°$ and $+6°$.

Contrary to what is expected from theory, variation of suction velocity has an appreciable effect on the lift. This effect is probably caused by variation of the relative quantities flowing above and below the flap, with change of suction velocity.

(*c*) *Effect on Lift of Unequal Suction Flows above and below the Flap.* Equation (1) of 2.3(*b*) shows that the increase in C_L, due to the differing suction flows above and below the flap, is proportional to $\bar{C}_p$ and to $\cot\theta_1/2$, and inversely proportional to U_s/U_0. The experimental values of $\bar{C}_p$ in the slot, given in 4.3(*d*), were used to determine the variation of ΔC_L with the test variables. The values of ΔC_L obtained in this way were compared with the values found from the experimental values of C_L. Both the values of $\bar{C}_p$ in the slot and the experimental values of C_L were found for the mid-span of the aerofoil.

Generally, the values of ΔC_L obtained from the experiments were much larger than those found from theory, although at $\alpha = -3{\cdot}0°$ the results obtained by the two methods were nearly identical.

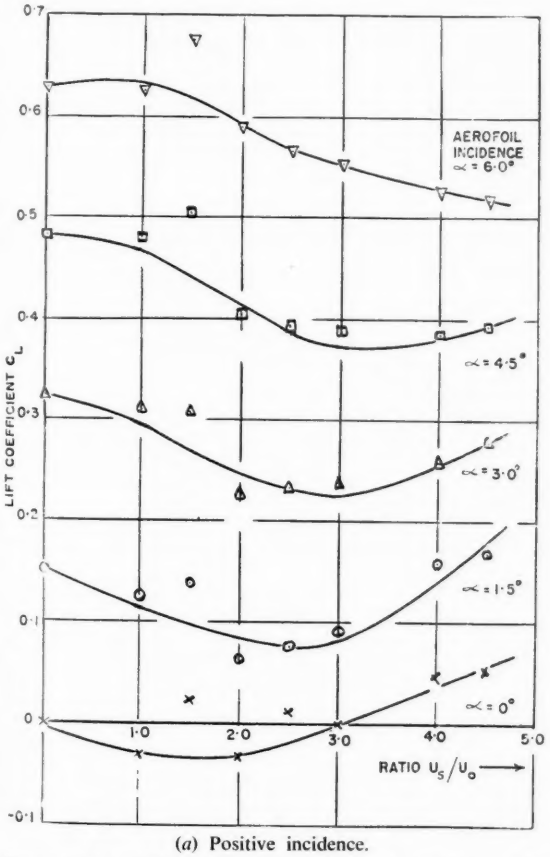
4.5. FORCE MEASUREMENTS BY STRAIN-GAUGE BALANCE

These measurements were made primarily to investigate the lift on the aerofoil when the flow was unstable. Because the flow was then of a three-dimensional nature, pressure measurements at mid-span could not be used to determine the lift.

It was hoped that the balance measurements would also give some information about the load on the flap. This was not found to be possible, however, and the average lift coefficients obtained from these measurements were therefore based on a 10 in. chord. A transformation of the aerofoil and flap to a flat plate enabled the approximate load on the flap to be determined theoretically. Integration of the load distribution on the flat plate was carried out from the leading edge ($\theta = 0°$) to the position of the rear sink on the combined circle plane ($\theta = \pi - \theta_1$ on Plane 2 of Fig. 4). This result was then compared with the complete integration to the trailing edge for the five adopted flap positions. The results of these integrations are given in Table IV and the load on the flap is seen to be small.

Attempts to measure the drag by the balance were disappointing, because the correction to be applied for the sink drag was relatively large. The sink drag coefficient C_{Ds} is equal to $2C_Q$, and $C_Q=0{\cdot}05$ at $U_s/U_0=2{\cdot}0$. Thus at $U_s/U_0=2{\cdot}0$, 0·10 had to be subtracted from the drag coefficient determined by the balance to obtain the aerodynamic drag, since the momentum of the suction stream was completely destroyed at the outlet of the pump. In many cases it was found that the resulting aerodynamic drag was negative, and it is realised that a lower stiffness of the strain-gauge elements in the drag direction is required for this type of work.

Results: (*a*) *Lift and Pitching Moment.* Fig. 21 shows lift curves at $U_s/U_0=2{\cdot}0$ for flap positions from $0{\cdot}150c$ to $0{\cdot}025c$, as obtained from the balance measurements and from pressure plotting at mid-span. The results obtained by the two methods were of the same form when the flow was stable, but differed considerably when it was unstable. In addition, the lift curves for the three outer flap positions (with stable flow) and for the aerofoil without suction were extended until the aerofoil stalled from the nose (Fig. 22). As the flap was retracted, the stalling angle of incidence increased and $C_{L\max}$ decreased.



(*a*) Positive incidence.

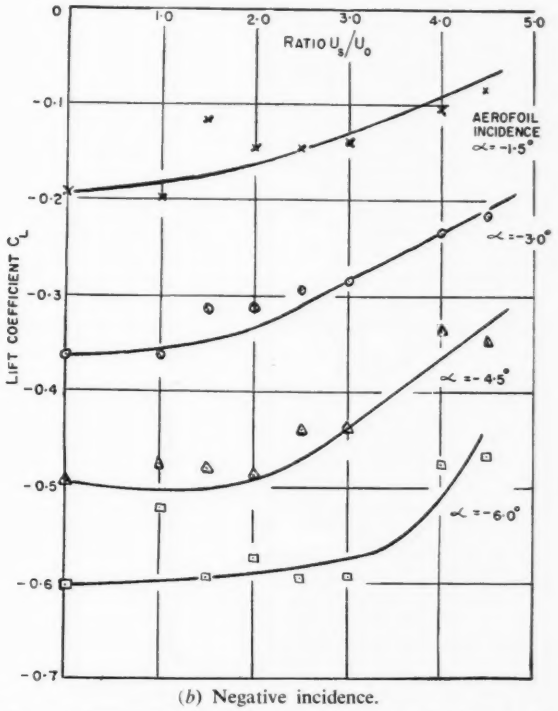
FIGURE 20 (*a*). Effect of suction velocity ratio U_s/U_0 on mid-span lift coefficient C_L. Flap at $X/c=0{\cdot}100$ from rear of aerofoil. Stable flow. Lift determined by pressure plotting at mid-span of aerofoil.

TABLE IV

Ratio of distance of flap from rear of aerofoil to chord, X/c	*Ratio of lift coefficients (correct value to pressure plotting result)*	*Approximate percentage increase of C_L from pressure plotting due to load on flap*
0·025	1·0001	0·01
0·034	1·0002	0·02
0·050	1·0018	0·18
0·100	1·0092	0·92
0·150	1·0219	2·19

Figure 23 shows the position of the centre of pressure on the aerofoil and the pitching moment coefficient about the quarter-chord point. Fig. 24 shows the relation between $C_{Mc/4}$ and the average value of C_L determined by the balance. In general, the behaviour of the aerofoil without suction was similar to that obtained with suction at $U_s/U_0=2{\cdot}0$, the flap being situated at $X/c=0{\cdot}100$ in both cases.

(*b*) *Lift with Lower Half of Slot Closed.* The aerofoil was also tested on the strain-gauge balance with the lower half of the slot closed and the flap extending to $X/c=0{\cdot}0625$ (stable flow), as described in 3·2(*a*)(6). The average lift coefficient for the aerofoil with this modification is plotted against incidence and suction-quantity coefficient in Fig. 25. The lift curve at $X/c=0{\cdot}100$ and $U_s/U_0=0$ (obtained from the balance) is included for comparison.



(*b*) Negative incidence.

FIGURE 20 (*b*). Effect of suction velocity ratio U_s/U_0 on mid-span lift coefficient C_L. Flap at $X/c=0{\cdot}100$ from rear of aerofoil. Stable flow. Lift determined by pressure plotting at mid-span of aerofoil.

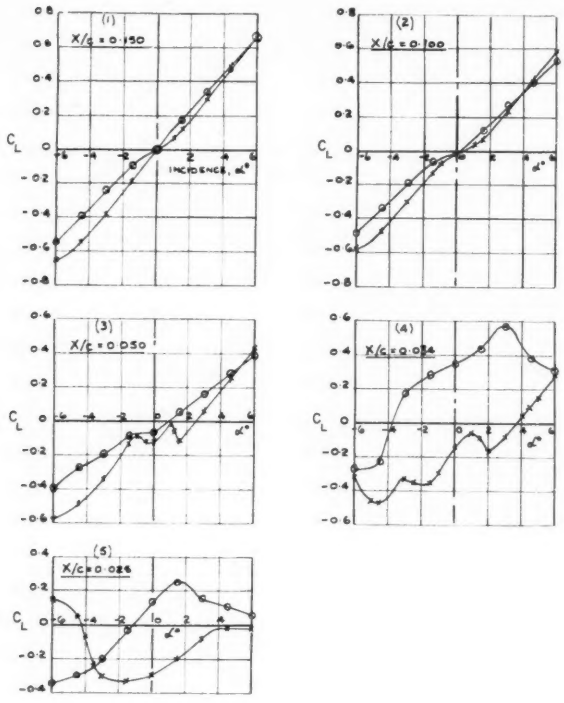


FIGURE 21. Comparison of lift curves.

4.6. WAKE-TRAVERSE EXPERIMENTS AND DRAG COEFFICIENTS

During stable flow when the flap was wound out from the aerofoil, measurements in the wake were difficult because of the very low total-pressure loss and the narrowness of the wake from the flap. However, with the flap retracted to produce instability, a very large wake of high total-pressure loss was found.

Wake traverses were made at mid-span with the flow unstable, measurements of static pressure being confined to a traversing plane well downstream, where the flow was nearly parallel to the free stream. To get some idea of the growth of the wake during instability, measurements of total pressure were made at three positions in the wake; the edge of the wake is outlined in Fig. 26.

The results of the wake traverses are expressed in Jones's[11] notation as $dC_{Dw}/d(y/c)$ where y is measured across the wake from the centre-line of the tunnel. The static pressure loss "p" behind the aerofoil was negative in all cases except for stable flow with suction, which is a rather surprising result.

Results from Experiments. Fig. 21 shows a wake traverse at $1{\cdot}175c$ from the aerofoil, for stable flow with the flap at $X/c=0{\cdot}150$. This profile shows how very small the losses in the wake were when the boundary-layer air was removed at the suction slot.

Variations in the wake readings were very large during instability and the results recorded were necessarily time-mean values. Several wake profiles for unstable flow are given in Fig. 27 and it will be seen that all of these are asymmetric at $\alpha=0°$.

The effect of suction on the conditions in the wake is shown in Fig. 28. At zero suction the wake was symmetrical and steady, in contrast to the violent disturbances found with suction when the flap was withdrawn. With the suction supply in operation, the traverses were completed at one tunnel speed to avoid scale effects. The effect of increase in suction during unstable flow is shown as an increase in the wake-drag coefficient.

The wake-drag coefficient C_{Dw}, determined by integration of curves such as those in Figs. 27 and 28, is plotted against (*a*) α, (*b*) X/c and (*c*) U_s/U_0 in Fig. 29.

When the flow was unstable, the values of C_{Dw} were very high and much greater than those obtained for ordinary thick aerofoils. The tunnel interference correction[12] to the drag due to a 25 per cent. thick aerofoil and the appropriate wake amount to 30 per cent. at $C_{Dw}=0{\cdot}4$ and 37 per cent. at $C_{Dw}=0{\cdot}5$. Consequently, the wake-drag figures found from this investigation are assumed to give only the order of magnitude of the wake losses.

It should be noted that the small wake-drag, for stable flow with the flap placed well downstream, could only be obtained by applying suction. There was then a sink drag coefficient equal to $2C_Q$. In practice, however, this sink drag would be recovered by ejecting the air removed by suction, at the velocity and pressure of the free stream.

4.7. TRAILING EDGE EXTENSION STRIPS

Regenscheit[6] found that an unstable flow, with a five-fold increase of profile drag, occurred when the "chords" of the upper and lower surfaces of a N.A.C.A. 23015 aerofoil with a rear suction slot were either equal or nearly equal. The instability was not explained, but it was similar to the unstable flow occurring with a free stagnation point, and was found when the suction flow

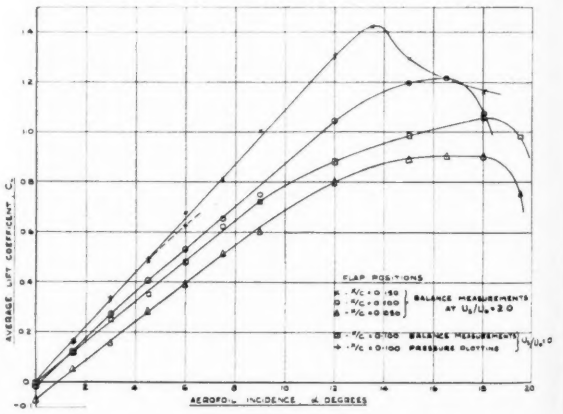


FIGURE 22. Lift curves for aerofoil up to stalling angle. Results by strain gauge balance at $U_0=120$ ft./sec. (based on 10 in. chord).

was high but not when the "chords" of the upper and lower surfaces differed considerably.

Some tests were therefore made with the ordinary 0·012 in. thick flap removed from the slot, and the hardwood fairing strips attached to the rear of the lower surface (Section 3.2(*b*)). The chord of the largest extension strip was 0·0451*c* and stable flow was found with it at $U_s/U_0 = 2{\cdot}0$. Separation occurred, however, at the rear of the lower surface, because the rear stagnation point occurred on the extension strip. During this stable flow there was a marked inflow on the upper surface and an outflow on the lower surface, showing that there was trailing vorticity along the span of the aerofoil.

As the suction flow was increased, the flow became unstable and similar to that obtained when the flap was retracted into the unmodified aerofoil. The cause of the instability was the release of the rear stagnation point into the free fluid. The value of U_s/U_0 at the onset of instability with this trailing edge strip was found to be 4·0. This experimental point is plotted in Fig. 3 (U_s/U_0 against X/c) and is found to lie close to the curve giving the position of the rear stagnation point as found by the tuft method. It should be remembered, however, that the aerofoil was asymmetric when fitted with the trailing edge strip.

With the next extension strip of chord 0·0324*c*, all the flow conditions were found to be the same as for the larger strip. The value of U_s/U_0 at the onset of instability with this strip was found to be 2·86, and this again agrees well with the experimental curve of Fig. 3. With either an extension strip of chord 0·0192*c* or a wire of diameter 0·0125*c* fixed to the lower surface, it was found that stable flow was impossible for any ratio U_s/U_0 other than zero. Thus, with these strips, the stagnation points were always in the free fluid.

4.8. RELATED EXPERIMENTS USING THE EJECTION OF AIR

Much evidence has been presented to show that the instability observed in these experiments is caused by a free stagnation point. It was therefore decided to investigate whether other free stagnation conditions could produce instability. Experiments were made on a two-dimensional aerofoil with air ejected from a leading edge slot, and also on a tube placed parallel to the

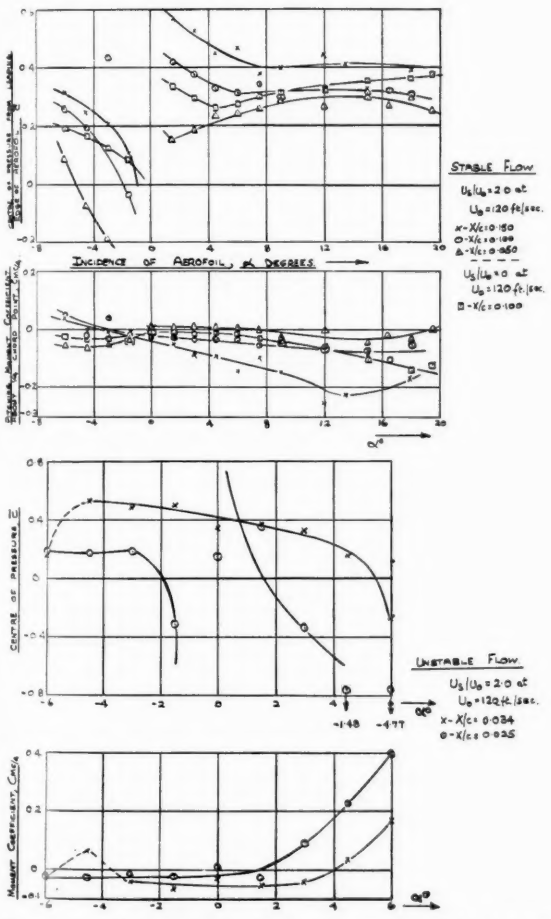


FIGURE 23. Variation of centre of pressure and pitching moment with aerofoil incidence during stable and unstable flows. Experimental results by strain gauge balance (based on 10 in. chord).

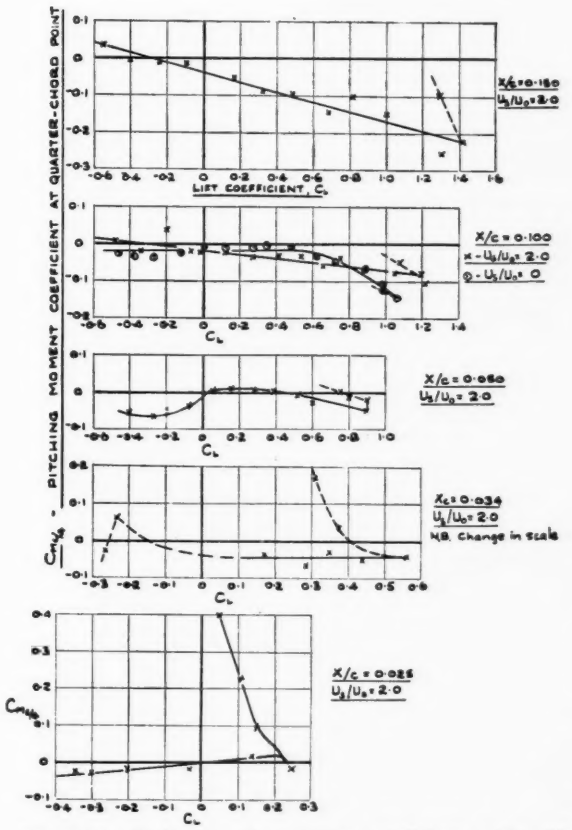
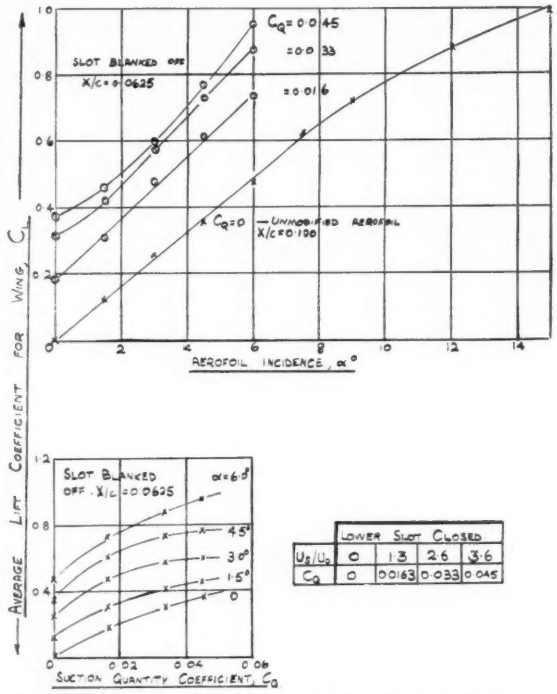


FIGURE 24. Variation of moment coefficient with lift coefficient for different flap positions. Results determined by strain gauge balance.



	LOWER SLOT CLOSED			
U_s/U_0	0	1·3	2·6	3·6
C_Q	0	0·0163	0·033	0·045

FIGURE 25. Effects of incidence and suction on average C_L when lower half of rear suction slot is closed. Results determined by strain gauge balance. Free stream velocity $U_0 = 120$ ft./sec.

undisturbed stream, with air ejected from its upstream end.

The Ejection of Air from an Aerofoil with a Leading Edge Slot. The apparatus for this experiment consisted of the "Lobster Pot" aerofoil rotated through 180° in the wind tunnel so that the slot was at the leading edge. The blowing air was provided by a small centrifugal fan. The flow was observed by the use of tufts, a streamer, and smoke.

When the stagnation front was about a chord upstream of the leading edge slot the flow was definitely unstable, and there was apparently an axial oscillation of the stagnation front. Thus the mode of instability was probably different from that previously found with the rear slot. When the stagnation front was brought nearer to the model, either by reducing the ejection velocity or by increasing the tunnel velocity, the flow was still unstable and could not be stabilised by the flap. In fact, when the flap was moved to the position of the transverse stagnation line, this line moved upstream ahead of the flap. This phenomenon is again different from that found with the rear slot.

At higher tunnel speeds the basic type of instability was a periodic one, with the flow inside the enclosing streamline changing from one surface of the aerofoil to the other, and yet maintaining an approximately two-dimensional form. Further increase in the tunnel speed made the flow three-dimensional, with a curved stagnation front upstream of the slot. When the stagnation front was very near to the leading edge of the aerofoil the flow appeared to be stable.

The Ejection of Air Upstream from a Tube in a Wind Tunnel. In all the aerofoil tests the model spanned the tunnel completely, so that the flow was probably affected by the boundary layers on the tunnel walls. Further experiments, avoiding this effect, were made by placing a 2 in. diameter tube on the centre line of a wind tunnel. The tube was connected to a blower, placed outside the tunnel, so that air could be ejected from the upstream end of the tube. The flow was observed by the use of tufts, a streamer attached to the mouth of the tube, and smoke.

With air ejected from the tube and with the tunnel air flowing, the stagnation point upstream of the tube was unstable. The instability consisted of an axial oscillation of the stagnation point, and at a large ratio of ejection velocity to free stream velocity the amplitude of this oscillation was about six tube diameters. As the ejection velocity was decreased, the amplitude of the oscillation decreased and the frequency increased.

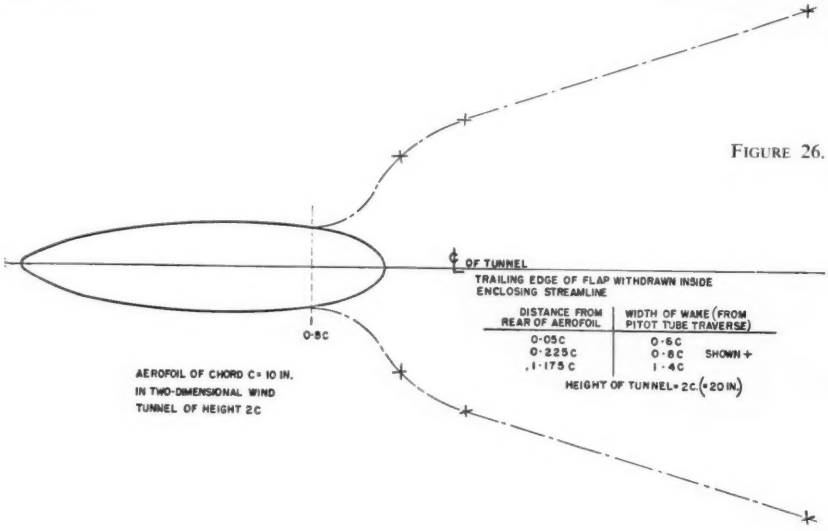

℄ OF TUNNEL
TRAILING EDGE OF FLAP WITHDRAWN INSIDE ENCLOSING STREAMLINE
DISTANCE FROM REAR OF AEROFOIL
WIDTH OF WAKE (FROM PITOT TUBE TRAVERSE)
0·05C 0·6C
0·225C 0·8C SHOWN +
1·175C 1·4C
HEIGHT OF TUNNEL = 2C (=20 IN.)
0·8C
AEROFOIL OF CHORD C = 10 IN. IN TWO-DIMENSIONAL WIND TUNNEL OF HEIGHT 2C

FIGURE 26. Growth of wake during instability.

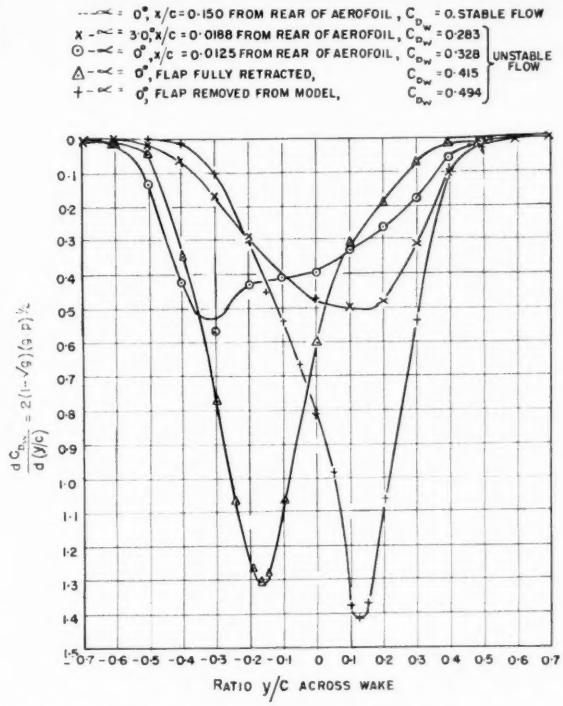

0°, x/c = 0·150 FROM REAR OF AEROFOIL, C_{D_w} = 0. STABLE FLOW
3·0°, x/c = 0·0188 FROM REAR OF AEROFOIL, C_{D_w} = 0·283
0°, x/c = 0·0125 FROM REAR OF AEROFOIL, C_{D_w} = 0·328
0°, FLAP FULLY RETRACTED, C_{D_w} = 0·415
0°, FLAP REMOVED FROM MODEL, C_{D_w} = 0·494
UNSTABLE FLOW
$\frac{dC_{D_w}}{d(y/c)} = 2(1-\sqrt{q})\sqrt{(q-p)}$
RATIO y/c ACROSS WAKE

FIGURE 27. Results of wake traverses at 1·175c from rear of aerofoil.
Effect of flap position, X/c.
Suction velocity ratio, $U_s/U_0 = 2{\cdot}0$ at $R = 6{\cdot}4 \times 10^5$.

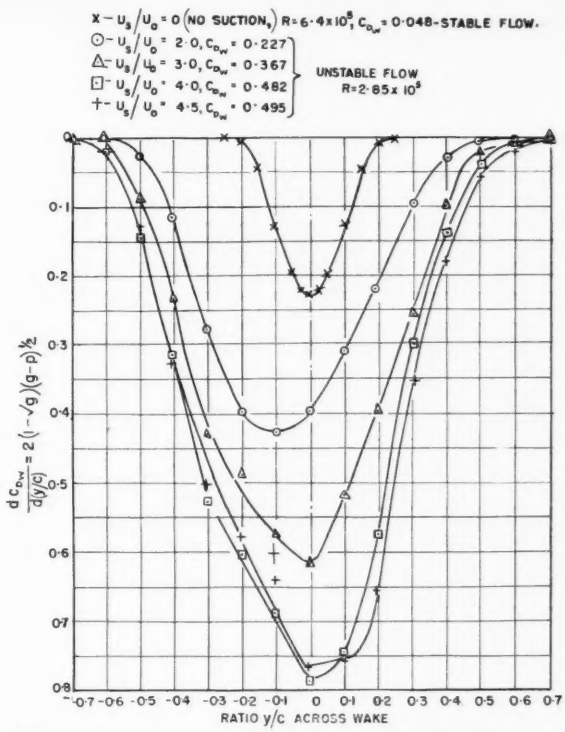

$X - U_s/U_0 = 0$ (NO SUCTION), R = 6·4 × 10⁵, C_{D_w} = 0·048 - STABLE FLOW.
U_s/U_0 = 2·0, C_{D_w} = 0·227
U_s/U_0 = 3·0, C_{D_w} = 0·367
U_s/U_0 = 4·0, C_{D_w} = 0·482
U_s/U_0 = 4·5, C_{D_w} = 0·495
UNSTABLE FLOW R = 2·85 × 10⁵
$\frac{dC_{D_w}}{d(y/c)} = 2(1-\sqrt{q})\sqrt{(q-p)}$
RATIO y/c ACROSS WAKE

FIGURE 28. Results of wake traverses at 1·175c from rear of aerofoil.
Effect of suction velocity ratio, U_s/U_0.
Aerofoil incidence, $\alpha = 0°$. Flap 0·0188c from rear of aerofoil.

An attempt was made to stabilise the flow by mounting a $\frac{1}{4}$ in. diameter rod on the centre line of the tube, projecting upstream from the outlet. It was found that the flow was still unstable with this rod in position, the stagnation point oscillating axially along the rod.

5. Discussion of Results

5.1. STABLE FLOW

When sufficient suction was applied to the rear slot of the aerofoil to remove all the boundary-layer air it was essential, for stable flow, to place the flap with its trailing edge beyond the enclosing streamline. In fact, the inconclusive nature of the earlier test on a "Lobster Pot" section by Preston and Gregory at the N.P.L. (unpublished) is now explained, since a flap was not then fitted in the slot and flow was unstable.

The actual suction flow required to produce favourable pressure gradients over the aerofoil was 49 per cent. higher than the theoretical flow. This increase in suction was apparently due to the effect of the boundary layers. Thinner "Lobster Pot" sections require smaller suction quantity coefficients, but of course these thinner sections will also be affected by the boundary layers.

When stable flow was obtained by suitable placing of the flap, the flow was found to be two-dimensional except in the tunnel-wall boundary-layers.

During stable flow, with the flap extending just beyond the enclosing streamline, trailing vorticity was observed behind the aerofoil, showing that there was a spanwise variation in circulation. Change of flap position or variation of suction flow altered the lift, mainly because of the different suction flows entering the slot above and below the flap.

Tests on a porous circular cylinder by Pankhurst and Thwaites[13, 14] showed that the flow could be stabilised by a very short flap normal to the rear surface of the cylinder. The formation of a wake behind the cylinder could be prevented by applying sufficient suction; the cylinder could therefore be used as a "disturbance free" smoke emitter.

5.2. UNSTABLE FLOW

Retraction of the flap into the suction slot brought the trailing edge of the flap within the enclosing streamline. Then, instead of the stagnation point existing in space in a stable manner, as had been assumed in Thwaites's scheme, a large and violent instability occurred. That the instability was associated with the free stagnation point has been proved beyond doubt.

The lift did not remain constant as the flap was retracted because a large wake was present during instability and vorticity escaped downstream. In fact, a large change in lift was produced as the flap was moved. The sectional and average lift coefficients differed considerably during unstable flow, due to the large variation in lift across the span of the aerofoil.

Usually, the application of suction has a favourable effect on the performance of an aerofoil, but here the totally unexpected result has been obtained that boundary layer suction at a rear slot without a flap produces an instability, with serious effects on the flow. However, Pfenninger[15] discovered by experiments in a water channel, that the rear stagnation point produced by suction at the trailing edge of an aerofoil would not remain fixed in space, although he overlooked the violence of the instability.*

While the "Lobster Pot" was under construction Thwaites[13] tested his proposal of flap withdrawal from a porous cylinder and found that the flow slowly went unstable. This result differs from the present case with concentrated suction, where the flow immediately went unstable when the flap was retracted. Similarly, Gregory[16] reported the existence of a wake and instability when the flap was removed from a 33 per cent. thick aerofoil with distributed suction over the last fifth of the chord. It is now known from the present investigation that the instabilities found with the porous cylinder and the 33 per cent. aerofoil were due to the presence of a free stagnation point. Thwaites and Pankhurst[13] stated that the unstable flow was essentially a problem concerning boundary layers, as they found that the flow was stabilised to some extent by a leading edge flap. In the present tests, however, a leading edge flap on the aerofoil made no difference to the instability.

The necessary length of flap for stabilisation of the flow about these aerofoils, with slots or distributed suction, is determined solely by the distance of the rear stagnation point from the surface. In one test, Thwaites found that the flow past the cylinder was stabilised by a wire of 0·028 in. diameter while the distance of the stagnation point from the cylinder could be shown theoretically to be approximately equal to the radius of that wire. If Thwaites had investigated the effect of large suction velocity ratios with the 0·028 in. wire in place, he presumably would have found that the flow went unstable and the cause of the instability would have been determined.

The difference in the sink strengths above and below the flap has been noted throughout this investigation. This phenomenon may have some effect on the development of the instability, which could be due to a coupling between the flap position and the differing sink strengths or the circulation appropriate to the differing sinks. In general, the upper surface sink was larger than the lower one; this moved the stagnation point below the chord line of the aerofoil and therefore increased the circulation. Apparently this process cannot go on indefinitely, and when a certain value of the circulation is reached, the flow breaks away and the operation starts again.

It has also been stated that the movement of the flap alters the distribution of suction at the rear slot, and hence adds to the complexity of the problem. When the flap was nearly within the aerofoil, the flap arched to and fro and this curvature would affect the sink

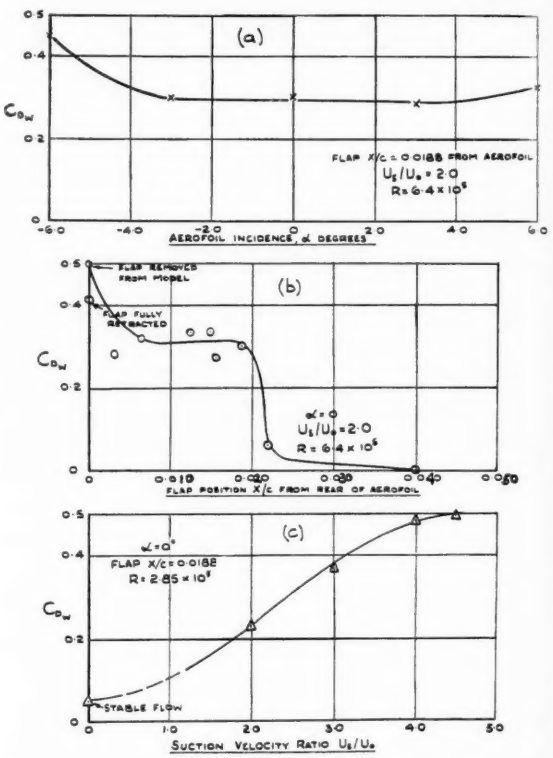


FIGURE 29. Effect of α, X/c and U_s/U_o on wake drag coefficient C_{D_W} obtained from traverses of the wake during instability.

strengths because of the different areas available for suction. Thus, under these circumstances, the coupling between flap position and circulation would be erratic, and this probably explains the complex unstable flow patterns.

With the porous circular cylinder, the sinks are distributed around the whole surface. Thus they do not have such a marked effect on the coupling with the flap position as in the case of the aerofoil with a slot. The state of the flow with perfectly symmetrical suction is open to conjecture, although it seems likely that such a suction flow would be impossible to achieve in practice.

Investigation of the stagnation front produced by a jet of air ejected against an oncoming stream also showed the presence of an instability, consisting mainly of axial movements of the stagnation front. It was found that this ejection flow differed from the suction flow about the aerofoil, in that the flow was not stabilised by a solid surface at the position of the stagnation line. The coupling effect discussed in connection with the unstable suction flow cannot explain the behaviour with ejected air because there are no differing ejection streams. In this case, it seems likely that the explanation of the unstable flow depends on the length of the jet ejected from the slot differing from moment to moment.

Potential flow theory also shows that a free stagnation point (line) is produced by rotation of a circular

*Dr. Pfenninger states that Ackeret has also investigated this instability.

cylinder at a peripheral speed of at least four times the free stream velocity. Experiments[17, 18] on rotating cylinders showed, however, that as the speed of the cylinder was increased a strong wake of reduced width was formed and there was a large increase of drag.

6. Conclusions

(1) With a flap protruding beyond the enclosing streamline and a suction flow at the rear slot greater than the theoretical amount, favourable gradients were produced over both surfaces of the aerofoil within the design incidence range, and no measurable wake existed behind the aerofoil.

(2) Retraction of the flap into the rear slot did not maintain constant circulation but set up a three-dimensional instability, involving a large wake drag. This was due to the presence of a free stagnation point, as with the porous circular cylinder.

(3) The lift curves for the aerofoil during the unstable flow were very unusual, and the tests showed that a large amount of control could be obtained with the application of only small forces, by using a movable flap in a rear suction slot.

(4) During stable flow, the sectional lift coefficients from pressure plotting were slightly less than the average values determined by the balance. When the flow was unstable, the pressure plotting and balance results differed considerably, although both may well be correct because of the three-dimensional nature of the instability and the presence of trailing vorticity caused by lift variations along the span.

(5) A free stagnation point may be of value in using the large drag obtained with it as a "wake brake." The effects of the unstable flow would be catastrophic unless the slots were of restricted length. Placing the slots at the roots of the wing would be best for aerodynamic reasons as well as for connection to the entry ducts of a gas turbine. The retractable flap would have to be wound out for stable flow when the "wake brake" was not required, so that no additional profile drag would then be caused.

ACKNOWLEDGMENTS

The author is indebted to Dr. J. H. Preston who supervised the research reported in this paper. The research was made possible by a D.S.I.R. Maintenance Grant.

Mr. N. B. Surrey constructed the aerofoil and Mr. F. M. Burrows the balance.

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Structure Weight*

by

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1. Introduction

This paper is primarily concerned with structure weight as related to aircraft shape and size, rather than with the saving in structure weight that may be brought about by improved methods of construction. The difference in weight due to differences in methods of construction is a question of structural efficiency, and this will not be discussed here. Structure weight, even when expressed as an overall percentage, is not in itself a measure of structural efficiency. If the depth of an aircraft wing were doubled, without altering the lift distribution on the wing, the bending loads would be unaltered so that the resultant end loads in the top and bottom surfaces would be halved. If the second wing had the same weight as the first the structural efficiency would be less. Conversely, if they both had the same structural efficiency the deeper wing would have the lower structure weight.

Comparison will be made of various groups of aircraft designed with the same skill at structural design; this means that they have the same structural efficiency. Also, as far as possible the same skill in design for aerodynamic properties in each group, the same skill in power unit design and the same skill in equipment design will be assumed. Thus for each group of aircraft it is being assumed that they have fixed structural, aerodynamic, power unit and equipment efficiency.

Design efficiency is regarded as the skill with which the designer combines the structural, aerodynamic, power unit and equipment properties. At the project stage of design a wide range of aircraft forms can be examined systematically and an optimum deduced, but the optimum so deduced must depend markedly on the basis used for determining it. The real optimum aircraft must be the one that will just meet the specification after the minimum expenditure of effort. This may be described as the one that costs the least. The problem, which always arises, is how to determine total cost. Frequently it is such a complex problem that it is avoided by deciding that some overall property, such as all-up weight or purchase price, is a sufficiently good indication of total cost. The total cost is considered here more closely but in general terms an attempt is made to break it down into total cost per lb. of structure, cost per lb. of power unit plus fuel, and cost per lb. of equipment at maximum take-off all-up weight. Thus the problem of selecting an optimum design of aircraft resolves itself into making the greatest use of those elements which cost the least. It must be appreciated that cheapness means total cost in the life of the aircraft and that this is not necessarily the same as low initial cost.

2. History

In the first days of flying the problem was merely to acquire sufficient speed to take off. This was done by obtaining an engine of definite size and power and by using engineering judgment to decide empirically the wing aspect ratio and wing loading and then making the structure as light as possible. Considerable knowledge of the effects of varying the different design properties is necessary before a project analysis on rigorous lines is possible. So the empirical approach had to be continued for a long time. This period extended to about 1925 during which time considerable knowledge of the various properties was being acquired.

In December 1925, Farren wrote a paper "The variation of the performance of an aeroplane with wing loading," R. & M. 994. In this paper he considers the effect of wing loading for a series of aircraft maintaining the same geometry. It is interesting to note that he compares first cost as well as top speed and useful load. His final analysis, however, compares the useful load as a percentage of the all-up weight maintaining a constant engine. For the purpose of his paper useful load includes all loads except the structure and the power plant, and he keeps the fuel unchanged for the different operating speeds. He deduces that the optimum wing loading for speeds of 80, 100 and 110 miles per hour are respectively 10·5, 15·3 and 18 lb./ft.^{2}

After the monoplane was introduced new experience had to be gained in designing aircraft, and it was not until January 1940 that Pugsley made a preliminary study of the structural and aerodynamic effects of wing thickness and fuselage size variation. The following remarks on structural weight and structural efficiency are from his unpublished work: —

> "The merits of the structure of an aeroplane can be judged in a number of ways. A simple and purely structural approach would be to regard the external geometry and loading of the aeroplane, and the size and disposition of its internal loads, such as bombs and fuel, as fixed; and then measure the efficiency of the structure by its weight. But this would set unduly narrow limits to the structural efficiency problem. A better aeroplane might result from changes of external geometry and a rearrangement of internal loading, alterations that would introduce aerodynamic as well as structural changes."

**A Lecture given to the Leicester Branch of the Society on 21st November 1952.*

He continues:—

"Any attempt thus to assess structural-cum-aerodynamic efficiency and to discuss structural arrangement is to move from the engineering science of strength and stiffness towards the art of aeroplane design."

In considering wings Pugsley takes three metal aircraft of then current design and, with the co-operation of the designers of the three aircraft, finds the effect on structure weight of changing wing thickness. With the definition of structural efficiency as it is used here it may be said that he retains the same structural efficiency throughout his investigations. He shows that the decrease in wing weight is 8·5 per cent., or 1·5 per cent. of the all-up weight, in changing from a wing thickness/chord ratio of 18 per cent. to 30 per cent. In one method of comparison he assumes that the weight saved is employed to increase the engine power and thus finds the top speed of the modified aircraft of the same all-up weight as the original. In this way he shows that the optimum root thickness/chord ratio is about 22½ per cent. but that the change between 18 per cent. and 30 per cent is very small, the total change in top speed being of the order of one per cent. In this study Pugsley has in mind the storage problem and recommends, as a result, that thickness/chord ratios of 30 per cent. could be used with advantage. One reason is that more fuel could be carried in the outboard portions of the wing, thus relieving the structural loads by these inertia loads.

Pugsley also considers fuselage weight and concludes that it appears that while method of construction has no substantial effect on fuselage weight, reduction in fuselage size leads definitely to reduced structural weight and aerodynamic drag. Continuing he states:—

"For full advantage to be taken of the variation of fuselage weight and drag with size it would clearly be necessary to transfer as much load as possible from the fuselage to the wings."

He quite evidently has this in mind when he recommends using wings of 30 per cent. thickness.

Fuselages are still made as small as possible, for aerodynamic drag and structure weight still both reduce with size. When the sizes and loads become such that fuselage weights increase with reduction in size, a compromise between structure weight and drag will be necessary. In the meantime it is sufficient to make this compromise only on wings.

All-up weight has continued to be regarded as the measure of efficiency for most project work since 1940. This can now be superseded, since much information has been collected by Masefield on the operating costs of aircraft and an estimate can be made at the design stage of the total cost of an aircraft for its whole life. In his lecture to the Royal Aeronautical Society in 1948 Masefield spoke on "Some economic factors in civil aviation." He was concerned with the efficiency of operating civil aircraft with special reference to the cost to the operator using aircraft as delivered to him. His cost figures are used here to predict how aircraft could have been designed to have a lower operating cost without any change in technical efficiency of structures, aerodynamics, power unit or equipment.

Since Masefield was interested in the aircraft as delivered, and since the majority had similar distributions of weight of fuel, power unit and structure for a particular stage length, there was no objection to his referring all costs to gross aircraft weight. For the present purpose this is not satisfactory because it would be assuming that the best aircraft is the one with minimum all-up weight. Discretion is used therefore in replacing all-up weight by structure weight or power plant weight where thought appropriate.

3. Design of an Aircraft with Discussion of Optima Values

No matter what parameter is to be used to define the optimum value of an aircraft it is essential that the specification requirements for the aircraft be known. Then families of aircraft, which must meet these requirements, can be found by varying the design properties at the designers' disposal. The specification requirements and design properties are given in Table I.

TABLE I

THE DESIGN OF AN AIRCRAFT

Specification Requirements	*Design Properties (Proportions at Designers Discretion)*
Load	Equipment
Range	Power
Speed	Aerodynamics
Height	Structures

As far as the specification is concerned the load which the aircraft has to carry may be taken to consist of passengers, freight or military load, together with such crew and equipment as is necessary for the flying of the aircraft. Thus the full design requirement is the carrying of a given load for a given distance at a given speed and height. The properties which the designer has under his control in meeting the requirements, are equipment, power, aerodynamics and structure. The object is to decide on some characteristic of the aircraft as being the measure of design efficiency and then to vary the design properties systematically to determine the optimum. Having decided the basis of prediction the definite optimum can always be determined, but the actual proportions of the various properties depend markedly on the basis of the prediction.

It was said earlier that the whole investigation was based on one of constant technical efficiency of the four properties—Equipment, Power, Aerodynamics and Structure. To do a complete investigation each of these would have to be varied and an optimum combination selected. This would be quite impracticable, but it is possible to take them in pairs and then by successive approximation to arrive at the optimum.

Table II gives the simplified properties that will be used to determine the optima. The equipment is

TABLE II

CONSTANT TECHNICAL EFFICIENCY

	Simplified Properties	
Equipment	Fixed by Aircraft Specification	
Power	Fuel consumption and thrust directly proportional to air density and power plant weight	
Aerodynamics and Structures	Empirical Formulæ (based on average properties of present aircraft) in terms of	
	Aircraft	Factor Speed
	Wing	Aspect Ratio Thickness/Chord Ratio Surface Area
	Fuselage	Dimensions
	Power Plant	Dimensions

assumed fixed by the aircraft specification. This is considered to be a reasonable assumption and, while there must be a relationship between the equipment and the other three properties, it seems so loosely connected that it would be unprofitable yet to attempt to balance its properties against any of the others. It must be remembered when arranging space for equipment that it has a definite volume as well as weight. The inter-relation of power, aerodynamics and structure is very great and must be considered. However, for the purpose of this investigation it is assumed that for the power plant and fuel "Fuel consumption and thrust are directly proportional to air density and power plant weight." No consideration is made of change in ratio of fuel consumption to thrust, with variation of general dimensions or speed of operation of the engine. This is a realistic view to take from the aircraft design standpoint, for the engines are supplied ready-made and to some extent have to be regarded as fixed. The remaining properties, aerodynamics and structures, are entirely at the aircraft designer's disposal and he can vary both over a wide range.

From a study of the average properties of past and present-day aircraft empirical formulae for aerodynamic drag and structural weight have been produced. The formulae are in terms of aircraft factor and speed, wing aspect ratio, thickness/chord ratio and surface area, fuselage dimensions and power plant dimensions.

Although only the optimum combination of aerodynamics and structures is being found, it seems that the work could be repeated with advantage to obtain a balance between the power unit and aerodynamics, to see whether the engines being provided are of the best form.

The technical parameters which can reasonably be used for the determination of the optimum aircraft are all-up weight, structure weight, power plant weight and fuel weight. The main influences of these parameters are:—

All-up weight affects take-off and landing and the transportation of the aircraft on the ground. There is also an upper limit to all-up weight on any particular runway because of the strength of the runway.

Structure weight influences the prime cost of the airframe and to a lesser extent the maintenance costs.

The power plant and fuel can be combined in the present analysis because of the basic assumptions that fuel consumption and thrust are each proportional to air density and power plant weight. Thus for a given speed and range the ratio of power plant weight to fuel weight will be constant. The combined power plant and fuel will have an initial cost that is entirely that of the power plant. The maintenance costs of the engine are a much greater proportion of the initial cost than those for the structure. The fuel costs are, of course, practically all running costs.

4. Determination of Various Aircraft Optima

Several aircraft optima will now be determined. The method is quite general, but for convenience an aircraft designed for civil or transport use will be considered. First a detailed description is given of how to predict an aircraft with minimum all-up weight. Then this aircraft will be regarded as the standard one and the properties of all other aircraft will be given in terms of its weight, aspect ratio and thickness/chord ratio. Various families of aircraft are considered, all of which carry the same load at the same height and speed and for the same range. For the purpose of the calculation a load of 20 per cent. of the standard all-up weight, a height of 40,000 ft., a speed of 600 m.p.h. and a range of 4,000 miles are taken.

Figure 1 gives a family of curves showing the increase in all-up weight from the standard with changes in aspect ratio and thickness/chord ratio from their standards. Looking at these curves from left to right, it may be seen that as aspect ratio is increased the all-up weight is reduced to a minimum and then rises again. The extreme left curve is for a thickness/chord ratio of 0·9 of the standard; this reduces to a minimum at just

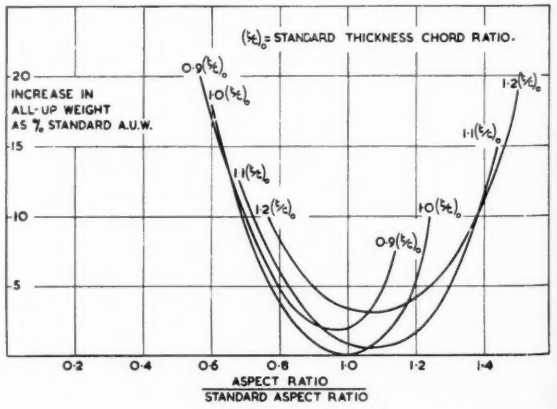


FIGURE 1. Increase in all-up weight with changes in aspect ratio and thickness/chord ratio from the standard.

Load 20 per cent. standard A.U.W. Range 4,000 miles. Speed 600 m.p.h. Height 40,000 ft.

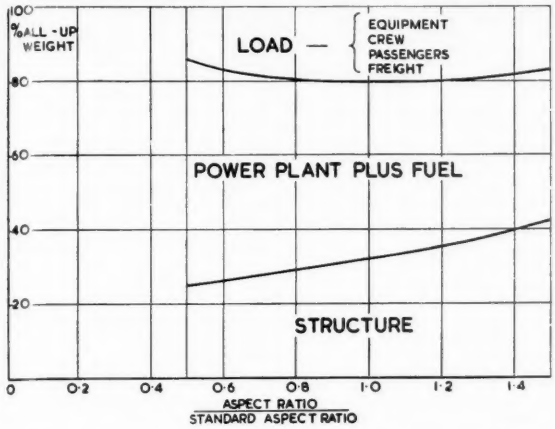


FIGURE 2. Distribution of weight of aircraft of minimum all-up weight at each aspect ratio.

Load 20 per cent. standard A.U.W. Range 4,000 miles. Speed 600 m.p.h. Height 40,000 ft.

under the standard aspect ratio, and thereafter increases very rapidly. The standard thickness/chord ratio curve becomes better than the 0·9 one at about 0·64 times the standard aspect ratio, and it reaches a minimum at the standard aspect ratio. The curve for 1·1 times the standard thickness/chord ratio becomes better than the one with standard thickness/chord ratio at an aspect ratio of 1·06 times the standard. This curve reaches a minimum at about 1·08 times the standard aspect ratio. The 1·2 times standard thickness/chord ratio becomes better than the 1·1 at about 1·38 times the standard aspect ratio.

The envelope of all these curves gives the aircraft of minimum all-up weight at each aspect ratio, the optimum thickness/chord ratio being chosen in each case. The remainder of this discussion is concerned only with this envelope curve, so that whenever an aircraft of a particular aspect ratio is mentioned it may be assumed that the appropriate optimum thickness/chord ratio has been chosen. Fig. 2 gives a distribution of the weight of each aircraft of minimum all-up weight at each aspect ratio. This distribution is given in terms of the percentage all-up weight of each aircraft and not as a percentage of the standard all-up weight. The weight of each aircraft is broken down into three parts, structure weight, power plant plus fuel weight and load which includes equipment, crew, passengers and freight. As the load is the same for each aircraft it means that the aircraft with minimum all-up weight is the one with maximum percentage load. As can be seen from the figure, the load is 20 per cent. of the all-up weight for the standard aircraft reducing to 14 per cent. at half the standard aspect ratio and reducing to 17 per cent. at $1\frac{1}{2}$ times the standard aspect ratio. The remainder of the weight which is distributed between the power plant plus fuel and structure is very differently proportioned at these two extremes, the structure weight being 25 per cent. at an aspect of half the standard and 42 per cent. at $1\frac{1}{2}$ times the standard aspect ratio. It should be appreciated that most modern aircraft have been designed with structure weight percentages corresponding approximately to the aircraft with minimum all-up weight, so that, should radical changes be contemplated, special attention will have to be paid to other incidental changes, such as storage capacity, that may not immediately be foreseen.

If the aircraft is broken down into the three weights of structure, power and load, then an optimum could be deduced for any of the three. An optimum on load is identical with the optimum corresponding to minimum all-up weight. Fig. 3 shows these three optima. The top curve gives the aircraft of minimum all-up weight at each aspect ratio and the optimum is, of course, at the standard aspect ratio. The ordinate of all-up weight is given as a percentage of the standard all-up weight. The same ordinate scale, namely percentage standard all-up weight, is used for the other two curves. The second one is the aircraft of minimum power plant plus fuel weight at each aspect ratio and the optimum occurs at an aspect ratio of 1·3 times the standard aspect ratio, the power plant plus fuel weight at that aspect ratio being 45 per cent. of the standard all-up weight. The aircraft of minimum structure weight at each aspect ratio is shown in the lowest figure. The optimum occurs at an aspect ratio of 0·75 times the standard aspect ratio, and the structure weight at that aspect ratio is $29\frac{1}{2}$ per cent. of the standard all-up weight.

5. Discussion of the Optima Aircraft and Introduction of Cost as a Parameter

Figure 3 shows optima aircraft corresponding to minimum all-up weight, minimum power plant plus fuel weight and minimum structure weight. Further

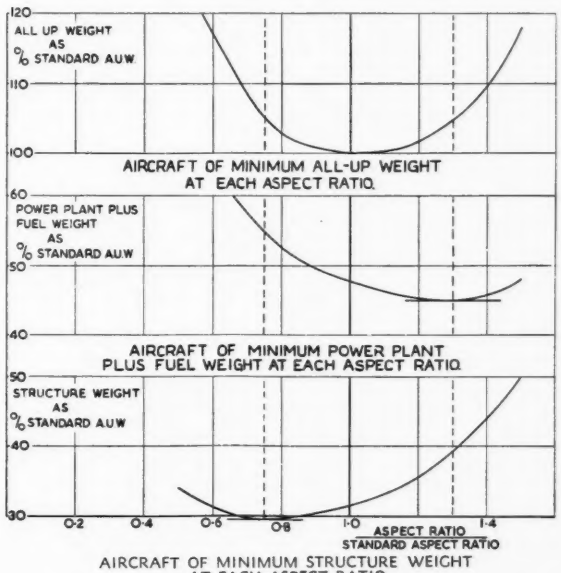
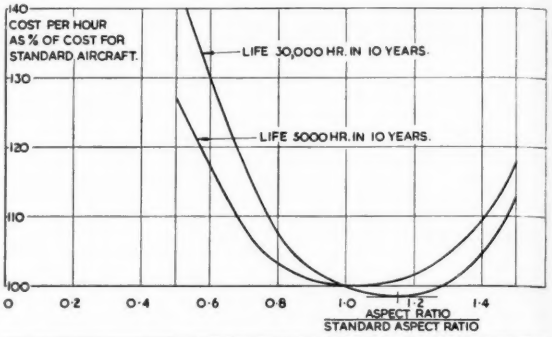


FIGURE 3. Various optima aircraft. Load 20 per cent. standard A.U.W. Range 4,000 miles. Speed 600 m.p.h. Height 40,000 ft.

information is needed to decide which of these is the best aircraft, and whether there is a better aircraft than any of them. Cost is now introduced as a parameter linking all-up weight with cost, power plant plus fuel weight with cost, and structure weight with cost. To do this a further specification requirement is necessary, namely the length of time the aircraft is expected to operate, in order to give a balance between prime cost and operating cost. All-up weight influences landing speeds and so effects running costs. The power plant plus fuel weight is in two major parts, the power plant which has a fairly high initial cost and a moderate running cost, and the fuel which has a negligible initial cost and a high running cost. The structure weight has predominantly a high initial cost with only a small running cost. In making the comparison it has been assumed that the life of the aircraft is ten years. The difference which would be introduced by having a lower life is not great provided that the same number of hours are flown. The only difference, in fact, is the cost of insurance and interest on capital. For this particular aircraft design which is being considered the fuel weight is a high proportion of the power plant plus fuel weight and there are only running costs for fuel. Thus at a very low flying life when the average cost per hour of the structure is high the predominant cost of the aircraft is that of structure; the power plant plus fuel cost being correspondingly less. At a total life of 5,000 hours the cost per lb. of structure weight has dropped to exactly the same as the cost per lb. of power plant plus fuel weight. When the flying life is increased to 30,000 flying hours the cost per lb. of structure weight is only about 0·4 of the cost per lb. of power plant plus fuel weight.

Figure 4 compares the percentage change of cost with change in aspect ratio for aircraft with lives of 5,000 and 30,000 flying hours. The cost per hour is given as a percentage of the cost for the standard aircraft. The cost per hour of the standard aircraft with a life of 5,000 hours is, of course, appreciably higher than the cost of the standard aircraft with a life of 30,000 hours. These curves of relative cost show that the 5,000 hour aircraft has a minimum cost when the aspect ratio is equal to the standard aspect ratio. For an aircraft with a life of 30,000 hours the minimum occurs at an aspect ratio of 1·15 times the standard aspect ratio, the cost being reduced by 1½ per cent. from the cost of the standard aircraft. It is realised that this reduction in cost is small; but another important feature is that, as can be seen from the shape of this curve, the cost per hour increases rapidly as the aspect ratio reduces below the standard aspect ratio, so that if an aircraft were built with an aspect ratio appreciably less than the standard, the all-up weight would not be increased nearly so much as the cost per hour of flying the aircraft. In fact, it is common practice at the moment to build aircraft of about 80 to 90 per cent. of the aspect ratio which would give minimum all-up weight. The increase in all-up weight by so doing is only 1 to 3 per cent. but the increase in cost from the aircraft of minimum cost would be 5 to 10 per cent.

The table at the bottom of Fig. 4 gives the percentage structure weight and power plant plus fuel weight for



DESIGN FEATURE OF AIRCRAFT	ASPECT RATIO / STANDARD ASPECT RATIO	STRUCTURE % AUW	POWER PLANT PLUS FUEL % AUW.	FUEL WEIGHT WING VOLUME LB/CU FT.
MIN. STRUCTURE WT.	0·75	28	53	8·5
MIN. COST FOR 5000 FLYING HOURS.	1·00	32	48	8·5
MIN. COST FOR 30000 FLYING HOURS.	1·15	35	45	8·5
MIN. POWER PLANT PLUS FUEL WEIGHT.	1·30	38	43	8·5

FIGURE 4. Percentage change of cost with change in aspect ratio for aircraft with lives of 5,000 and 30,000 flying hours.

the various aircraft. It can be seen that for the aircraft with 5,000 flying hours structure weight is 32 per cent. and power plant plus fuel weight is 48 per cent. When the flying life is increased to 30,000 flying hours and structure weight becomes cheap in terms of cost per hour, the weight of structure becomes 35 per cent. and the power plant reduces correspondingly to 45 per cent. For comparison the aircraft of minimum structure weight and the aircraft of minimum power plant plus fuel weight are also given, the structure weights being 28 and 38 per cent. of the all-up weight respectively and the power plant plus fuel weight 53 and 43 per cent. of the all-up weight. The last column has been included to give an indication of the problem of stowage. Fuel weight divided by wing volume is an indication of the stowage efficiency. It can be seen that over the whole range from 0·75 times the standard aspect ratio to 1·3 times the standard aspect ratio, the fuel weight divided by wing volume remains constant at 8·5 lb./ft.[3]

In Table III a comparison is made of the structural cost and the power plant plus fuel cost for several types

TABLE III

COMPARISON OF STRUCTURAL COSTS AND POWER PLANT PLUS FUEL COSTS FOR AIRCRAFT OPERATION

Speed = 600 m.p.h. Life approximately 10 years (not critical). Costs include initial cost of aircraft but not administrative overheads.

Life: Flying hours	*Average Flight: miles*	*Cost (per flying hour)*	
		£ per 1,000 lb. Structure	*£ per 1,000 lb. Power Plant and Fuel*
30,000	4,000	0·52	½ (1·16) + ¾ (1·3) = 1·26
5,000	4,000	1·70	¼ (2·9) + ¾ (1·3) = 1·70
5,000	400	1·80	¼ (2·9) + ¼ (17·0) = 6·4
1,000	400	7·53	¾ (10·7) + ¼ (17·0) = 12·2

of aircraft. The two aircraft already discussed in the previous figures are given first, namely, aircraft with lives of 30,000 and 5,000 flying hours respectively, and both with a range of 4,000 miles. For comparison two other aircraft have been included, with flying lives of 5,000 and 1,000 hours, but having one tenth of the range, namely 400 miles. The costs given include the initial cost of the aircraft, but not administrative overheads. The speed of all the aircraft is taken to be 600 m.p.h. The life is taken as ten years but the actual value of the life is not critical to the calculation. It can be seen from this table that the cost per flying hour of structure increases from £0·52 per thousand lb. at a life of 30,000 hours to £1·7 at 5,000 hours. The corresponding cost per thousand lb. of the power plant plus fuel is £1.26 for 30,000 hours and £1·7 for 5,000 hours. For the aircraft with a shorter range the cost of structure for 5,000 hours life is slightly greater than the cost for the aircraft with a longer range. This is due to increased maintenance costs due to landing and take-off. The power plant plus fuel, however, is appreciably greater. This is due to the actual fuel cost per hour remaining the same, while the weight of fuel carried is reduced to approximately 1/10. Thus the cost per thousand lb. of power plant plus fuel is much greater. The study of comparative costs of power plants and fuel would be an interesting problem in itself, but it is outside the scope of the present talk. It is sufficient to note that on the assumptions made the short life aircraft with short endurance has a higher cost of power plant plus fuel relative to structure, just as the longer range and longer life aircraft has. This short-life short-range aircraft may be regarded as having characteristics similar to those of a fighter. The only type of aircraft which appears to have a structure cost up to the power plant plus fuel cost is that of an aircraft of 5,000 flying hours with an average range of 4,000 miles. This might be considered to correspond to a long range bomber. Thus for all aircraft, except the long range bomber, the cost of the power system is greater than the cost of the structure per lb. weight and the cheapest aircraft will be one with a greater structure weight than that of the aircraft designed for minimum all-up weight.

6. Return to History and Recommendations for the Future

Having demonstrated that the relative costs of the structure and of the power system depend on the number of hours the aircraft is intended to fly, it is interesting to retrace the development of the aircraft with special reference to cost. If an aircraft can have its average cost per hour reduced by increasing the structure weight and compensating this by a drop in the aerodynamic drag and the weight of the power system, it means that the structure was cheaper than the other commodities. Of course, this can only be continued until the minimum cost is reached; thereafter the structure becomes the expensive commodity. With this concept of cost, the whole development of aircraft is traced, taking into account changes in technical efficiency during the time the aircraft has been under development. Fig. 5 compares the cost of aerodynamics, power system, and

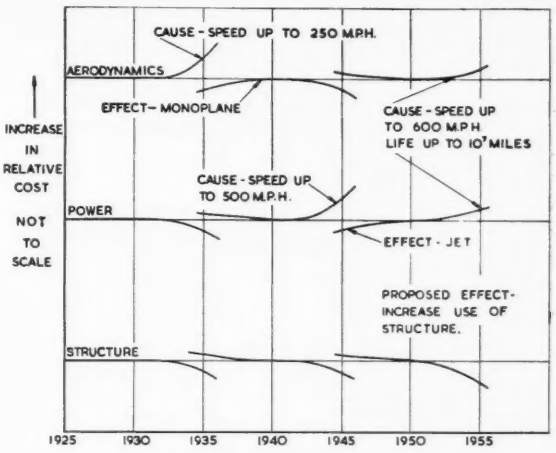


FIGURE 5. Distribution of aerodynamic power and structure costs in terms of optimum cost of each.

structures at different dates. The figure has been prepared without any attempt to include a scale to the costs. It is assumed that on average the optimum values of aerodynamics, power system and structure will have been used. It is almost certainly true that in time the optimum values will be obtained empirically, for if several prototypes are made to a given specification the aircraft nearest to the absolute optimum will be the best and will therefore be the one most likely to go into production.

In the first stage of the development of aircraft from, say, the Wright Brothers to the introduction of the modern cantilever monoplane, it was hardly possible to include numerical values of the relative merits of the three characteristics, and so the best would be chosen entirely on an empirical basis. As speeds increased up to the time at which the monoplane replaced the biplane, the cost of producing a satisfactory aerodynamic shape was increasing, and it was becoming more important to approach the ideal streamlined aircraft of Sir Melvill Jones. With the change to the monoplane there was a sudden drop in the cost of aerodynamics. It may be assumed that the first effect of changing over to a monoplane was to make aircraft with thickness/chord ratios as small as possible, compatible with maintaining a structure weight at about the same percentage of all-up weight that it had been for biplanes, that is 30 per cent. or a little over. The best advantage of low cost of aerodynamics could have been achieved by having a higher drag and correspondingly lower structure weight. This was encouraged by Pugsley some five to seven years later, in 1940, when he recommended that thickness/chord ratios up to 30 per cent. should be used. By increasing the thickness/chord ratio the structure weight was greatly reduced and designers proudly claimed that they had structure weights of less than 30 per cent.

It is suggested that by 1943 a new balance of aerodynamics, structures and engines had been achieved empirically.

Then the jet engine was introduced. This meant a revolutionary improvement in engine efficiency with a consequent reduction in cost to do a particular job. With this sudden reduction in cost of engines it was worthwhile to make more use of the engine at the expense of aerodynamics and structures. This was done gradually by increasing the total weight of engines and fuel. The fuel and power plant weight of a modern long range transport has risen to as much as 60 per cent. of the all-up weight. The time has now come that, with the speeds having increased still further, it has become more costly to produce satisfactory aerodynamics and also, for the long ranges, it has become more costly to provide the power system, and therefore on a comparative basis the structure is the cheapest of the three. Thus more structure weight is needed, but without loss of structural efficiency. Even for the fighter it is probably more efficient to use structure weights slightly greater than have been the custom in the past, but for the transport aircraft the situation is more serious.

7. Further Developments

It has been suggested that the time has now come when aircraft should be made with a greater percentage of structure weight; all this being maintained for a standard efficiency of structures, aerodynamics and engines. Also, there have been two enormous changes in aerodynamics and engines, the aerodynamics one in the early thirties and the engines one just after the Second World War. It seems that the time is ripe for a large step forward in structures, in addition to the suggested improvement in design, of using a greater structure weight without any change in structural efficiency. At present aircraft are made in such a manner that rather more than half their structure weight is made up by the weight of joints. It is considered that for many applications it would now be worthwhile to machine the structure out of a solid block of material. This has been done to some extent in fighter aircraft, and perhaps this is not really a prediction of what should happen in the future, but merely a statement that the time has already come in which it is worthwhile having this large increase in structural efficiency.

From rough calculations on what should happen if there is a marked increase in structural efficiency it is concluded that the structure weight should be left at the present optimum percentage, rather than reduced and replaced by fuel. Tackling this new era of structural efficiency now can avoid what happened in the past with the sudden change in aerodynamics, when about seven years after the change Pugsley was recommending using thicker wings and therefore wings with greater drag; again it took several years before designers would use jet engines for long range civil work and therefore larger proportions of fuel. This time it is essential right from the start, to try to maintain the percentage structure weight that is considered to be the optimum now and so to take full advantage of the improved efficiency of structures.

ACKNOWLEDGMENT

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TECHNICAL NOTES

This section of THE JOURNAL *is intended for short papers, comments upon published or unpublished papers with a limited circulation, interim results of research in hand, or notes for further research, or comments on notes already published, or letters raising technical points or asking questions. Contributions, which will be eligible for Journal Premium Awards, will normally be published within two months of being received.*

The Use of Radar to Measure the True Air Speed of an Aircraft

by

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THERE is at present no thoroughly satisfactory method of measuring accurately the true air speed of an aircraft, particularly at high speeds.

Ground radar can accurately follow a high-speed aircraft but, without the development described here, can only measure the speed of the aircraft relative to the ground. The method developed by the authors makes immediate allowance for wind speed and is capable of giving the true air speed of an aircraft to within one per cent. There is no apparent reason why the method should not be applicable even at very high speeds and there is no limit in aircraft altitude at which the measurement may be made.

METHOD OF MEASUREMENT

The method consists, fundamentally, in arranging for the aircraft whose speed is to be measured to drop at accurately known time intervals a series of bundles of metallic foil that reflect the radar signals. A measurement by the radar of the separation of the bundles then enables a determination of aircraft air speed to be made; and since the pattern drifts with the wind this measurement can be made more or less at leisure.

It is not necessary to track the aircraft itself at all accurately. However, measurements need to be made within half a minute of the dropping of the targets, otherwise errors are introduced by the natural variability of wind with both time and position. This variability is in fact the reason why a radar measurement of aircraft ground speed, supplemented by a separate measurement of wind speed at flight level by observation of a balloon, cannot yield a precise value of the air speed.

In practice it is found desirable to drop the bundles in a line running away from the radar set and approximately up or down wind. It also simplifies the job of the radar operator if, during the run up of the aircraft to the test point, other bundles of foil are dropped every few miles. These ensure that the radar will be ready to observe the test immediately the foil is dropped. They also enable a more accurate measurement of aircraft height to be made by the radar than is possible from the distant bundles.

Received 4th August 1953.

ACCURACY REQUIREMENTS IN THE OBSERVATIONS

Radar is usually most accurate in its determination of range; in particular changes of range to a target can be measured very precisely. Accuracy in measurement of elevation angle or azimuth angle is usually comparatively low. It is hence desirable to carry out measurements of aircraft speed in a manner that makes most effective use of these properties of radar. This implies that the aircraft be flown at a constant and known height and at a large distance from the radar, and this is the condition in which the technique is most readily applied. For example, the errors in different parameters that cause an error of 0·1 per cent. in the measurement of air speed are given in Table I as a function of the elevation angle from the radar to the aircraft.

TABLE I

	Elevation angle			
	5°	10°	15°	20°
Range differences: per cent	0·1	0·1	0·1	0·1
Time intervals: per cent	0·1	0·1	0·1	0·1
Nominal range: per cent	13·0	3·3	1·5	0·8
Elevation angle	0·6°	0·3°	0·2°	0·15°
Nominal aircraft height if range is known precisely: per cent	13·0	3·3	1·5	0·8
Change in aircraft height from nominal in feet per mile flown	60	30	20	15

Provided that the wind changed little with height and that elevation angles were also accurately measured, the technique could be developed for the measurement of speed in diving flight with the aircraft diving towards the radar station. It is not likely, however, that quite such an accurate measurement of air speed would be possible under these conditions.

In the experiments reported here an overall accuracy of one per cent. in aircraft air speed was desired. It is doubtful if it would be worth while attempting to make measurements to much greater precision since fluctuations of one or two knots in mean wind speed in quite short distances, which are commonly experienced at all altitudes, would render any such measurements of doubtful value unless only an average value is necessary.

Measurements were normally made when the aircraft was at a range of more than 15 miles from the radar and the angle of elevation was less than 12°. Under these conditions the height of the aircraft needed to be known only roughly (to within 500 ft.) and the aircraft needed only to keep to within 100 ft. of its nominal height for the errors due to these factors to affect the measurement by much less than one per cent.

EQUIPMENT USED

The radar equipment employed was a laboratory model used for upper wind research. Its sensitivity was such that it could follow a medium sized aircraft out to a range of 100 miles. It used a magnetron transmitter, operating on a wavelength of 10 cm., and supplying a peak power of 400 kw., in the form of two microsecond pulses at a recurrence rate of 400 per second, into a 6 ft. diameter paraboloid antenna. The accuracy in range was ±0·01 mile and in angle ±0·3°. Since it is only changes in range that need to be measured accurately it is likely that most precision radars of the gunlaying type would be quite satisfactory for the purpose. The radar antenna was pointed towards the target by means of hand-operated controls during the period of measurement, though it could be rotated continuously by means of a motor for the purpose of an initial search.

The metallic foil used as a radar target is in bundles, each about the size of a match-box weighing only 30 gm. and containing about 1,500 strips 5 cm. long and 0·3 cm. wide. The terminal velocity of fall of these strips is about 150 ft./min. Individual bundles are visible on the radar out to a range in excess of 50 miles.

The bundles of foil were dropped from the aircraft by means of a mechanical launching device that was triggered from an electrically-driven clock. This launching device was developed and constructed by the Aeronautical Research Laboratories of the Australian Department of Supply and Development. It is clear that any tests on high speed aircraft would require the use of some such mechanical launching system.

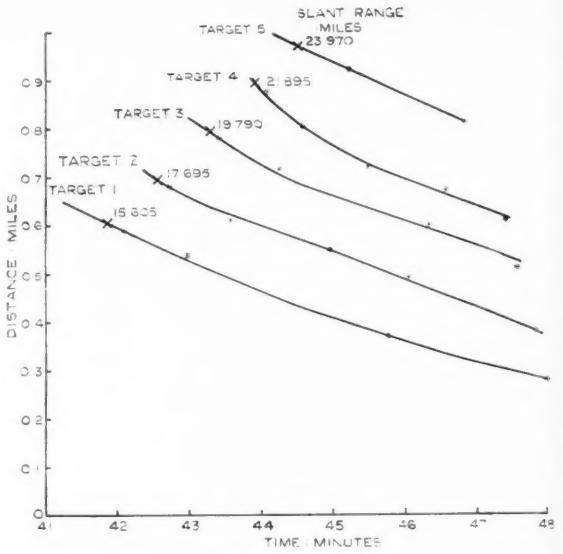

FIGURE 1. Paths of targets.

RESULTS

So far, only preliminary trials have been made of the method, and the results given here are intended mainly to illustrate the application of the method. For this reason the figures for one test are given in detail and the others briefly summarised. These results were obtained from tests on a Mustang aircraft flown at an indicated altitude of 15,000 ft. over the radar set in a line parallel to the mean wind, and targets were dropped when the aircraft was roughly 15 miles distant. The true altitude was measured by the radar when the aircraft was beginning its run and nearly overhead and found to be 15,900 ± 200 ft. In Table II are shown the times at which the targets were observed to drop from the aircraft and their subsequent positions.

The paths of the individual targets are plotted in Fig. 1 and are extrapolated backwards to the instant of dropping. The ranges at these instants are read off, as are the ranges of the preceding targets at the same times. From the difference between these two values and the known aircraft height and range the horizontal distance intervals are calculated. The accurately known interval between dropping the targets is used to calculate the aircraft true air speed as shown in Table III.

TABLE II

Target No.									
1 *Dropped* *41 min. 52 sec.*		*2* *Dropped* *42 min. 34 sec.*		*3* *Dropped* *43 min. 17 sec.*		*4* *Dropped* *43 min. 55 sec.*		*5* *Dropped* *44 min. 32 sec.*	
Time *Min. Sec.*	*Range* *Miles*	*Time* *Min. Sec.*	*Range* *Miles*	*Time* *Min. Sec.*	*Range* *Miles*	*Time* *Min. Sec.*	*Range* *Miles*	*Time* *Min. Sec.*	*Range* *Miles*
42 05	15·59	42 45	17·68	43 25	19·78	44 05	21·87	45 15	23·92
42 58	15·53	43 35	17·61	44 15	19·71	44 35	21·80	46 50	23·81
45 45	15·37	44 55	17·55	46 20	19·60	45 30	21·72		
48 00	15·28	46 00	17·49	47 35	19·51	46 35	21·67		
		47 50	17·38			47 25	21·61		

TABLE III

Targets	*Difference in slant range miles*	*Horizontal distance miles*	*Time interval seconds*	*True air speed m.p.h.*
2. 1	17·695 − 15·555 = 2·140	2·175	40·9	191·4
3. 2	19·790 − 17·635 = 2·155	2·183	40·6	193·6
4. 3	21·895 − 19·730 = 2·165	2·188	40·7	193·5
5. 4	23·970 − 21·805 = 2·165	2·187	40·9	192·5
5. 1	23·970 − 15·435 = 8·535	8·670	163·1	191·3
			Mean	192·5

In a similar manner results were obtained at a number of different air speeds. In Fig. 2 the difference between the mean radar true air speed (T.A.S.) and the indicated air speed (I.A.S.) as corrected for air density (on the assumption of an I.C.A.N. atmosphere) is plotted against corrected I.A.S. The figure shows results obtained when three or more measurements were available from four or more targets, and also the less reliable results when only two measurements were possible.

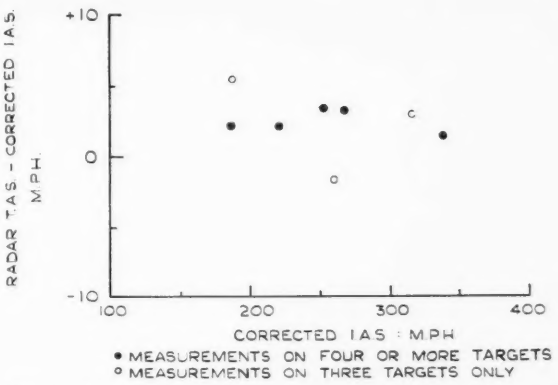

FIGURE 2. Variation of error in corrected I.A.S. with speed.

DISCUSSION OF RESULTS

The paths of the targets plotted in Fig. 1 show clearly the variability of the wind speed both in time and space. The mean wind on this occasion was only light, but similar magnitudes of change in higher wind velocities have also been found common. To what degree variability in wind speed is the cause of variability of the radar air speed is unknown. The indicated air speed as measured by pitot tube was observed at short time intervals and its variation compared with the variation in the radar measurements. While there was strong correlation on some runs, others gave no correlation at all. The reasons for this are not known, but it is considered unlikely that the radar measurements were in error by more than the one per cent. originally aimed at.

ACKNOWLEDGMENTS

The trials of the method were carried out in aircraft of the Aircraft Research and Development Unit of the Royal Australian Air Force with the active co-operation of the Aeronautical Research Laboratories of the Australian Department of Supply and Development. The latter body was responsible for the design and installation of the target launching device and recording equipment in the aircraft.

The Coriolis Effect

by

PAUL E. WYLIE

THE CORIOLIS EFFECT is a change in the motion of a body passing over the surface of the Earth due to the motion of the Earth itself. The effect may be manifested either as a horizontal acceleration, or, in the absence of the acceleration, as a deflection of the course. The acceleration, which appears in controlled courses such as those of aircraft, usually appears and is significant as a deflection of the vertical. This acceleration appears whenever a body, such as an aircraft, is forced to follow a great circle path over the Earth. The deflection of the course of a moving body appears alternatively whenever the body moves freely in its inertial path above the surface of the moving Earth. The most obvious example of the deflection effect is that of an artillery shell, which, in the northern hemisphere, always bears to the right in flight; another example is that of the Foucault pendulum, whose plane of vibration as time passes moves clockwise in the northern hemisphere and counter-clockwise in the southern. The artillery shell combines in its motion the motion of projection from the gun and the motion of the Earth about the polar axis as the latter motion exists at the site of the gun; the Foucault pendulum is not subject to any horizontal force and consequently tends to remain in its original plane of vibration and does so, except in so far as it is affected by the continuous change in the direction of gravity*.

According to Newton's second law of motion, whenever a moving body changes its speed, its direction of motion or both, it is certain that an acceleration is acting on the body. If the Earth were at rest an aircraft flying over it in any great circle at a constant speed and altitude would be subject to an acceleration towards the centre of the Earth, equal, according to Huygen's equation, to v^2/R, where v is the aircraft velocity

Received 29th April 1953.

*See "The Paradox of the Foucault Pendulum" by Paul E. Wylie, *Popular Astronomy*, Vol. LVII, No. 4, April 1949.

and R is the spherical radius of the Earth. This acceleration will act exactly in the direction of gravity, and will be manifested only by a small change in the weight of the aircraft. It is not the Coriolis acceleration, which acts in the horizon and not in the vertical; it is exceedingly small, as may readily be determined by substituting numerical values, and is introduced here only to eliminate it from further consideration.

Consider now an aeroplane flight, taking into account the rotation of the Earth. In general, it is easy to see that an aeroplane flying east in any great circle must, on account of the motion of its destination towards the east, caused by the Earth's rotation, introduce a "crabbing" or sidewise horizontal motion in connection with the forward motion of the aircraft. This curvature of path necessarily implies an acceleration in the horizon; it is the Coriolis acceleration. If, by manipulating the controls in the aircraft, the Coriolis acceleration is not introduced into its motion, then the aircraft, following its inertial path, will not reach its destination, but will arrive at a point to the right (in the northern hemisphere) of the destination, measured from the line of course.

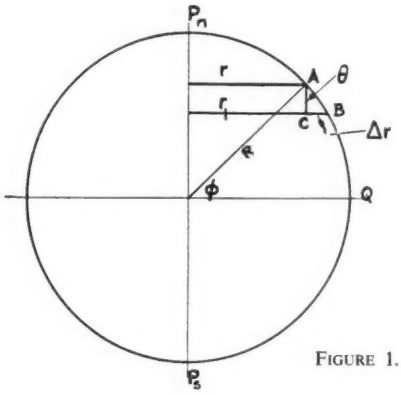

FIGURE 1.

The foregoing is obvious when the course is due south or due north; taking the case of a flight from north to south, and substituting a projectile for the aircraft, during the time of flight, the target will more farther, because of the greater radius about the axis of the Earth, towards the east, than will the gun, or the projectile, which shares only the easterly motion of the gun. It is equally easy to visualise the deflection for trajectories from south to north, the conclusion being the same. This conclusion is valid whatever may be the direction of the aircraft's course, or of the projectile's path. Before attempting its proof, an evaluation will be made of the Coriolis acceleration of a body constrained to move due south; that is, along a meridian.

Let the aircraft (Fig. 1) start at A, and move along the meridian of the Earth P_nAP_s towards Q, the equator, its velocity being v. The latitude is ϕ. Since the Earth is rotating, this course will involve an acceleration of the aircraft to the east (into the paper). At A, where the radius of the parallel is r, the linear displacement of A towards the east in unit time is ωr, where ω is the angular velocity, in radians per second, of the Earth's rotation. Similarly, at B, the linear displacement will be, in the same unit time, ωr_1. The difference, $\omega r_1 - \omega r$, or $\omega(r_1 - r)$, is the displacement, in unit time, towards the east, of B, relative to A. Twice this displacement will be the uniform acceleration that must be given to an aircraft in order that it will arrive at B; that is, $a = 2\omega(r_1 - r)$. If $(r_1 - r) = \Delta r$, then $a = 2\omega\Delta r$.

Returning to Fig. 1, Δr will be evaluated. Since displacements in unit time have been used, for consistency, make $AB = v$; that is, the displacement of the aircraft towards the south in unit time. Since v is small compared with R*, ACB may be considered to be a triangle without introducing sensible error. Then $CB = \Delta r$ and the angle $BAC = \phi$. Then $\Delta r = v \sin\phi$, and therefore $a = 2\omega v \sin\phi$, which is the expression for the value of the Coriolis acceleration of an aircraft moving south along a meridian. Exactly the same procedure could be followed for an aircraft flying north along a meridian, and the resulting acceleration would be found to be the same in magnitude and relative direction (to the left in the direction of flight, in northern latitudes) in this case. In the southern hemisphere the acceleration is the same

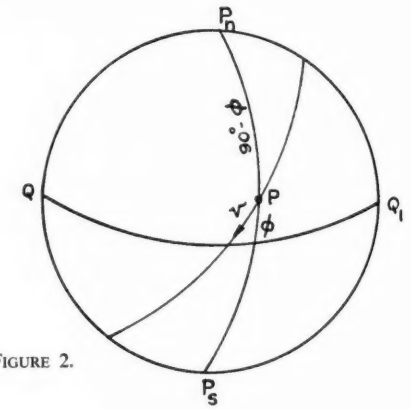

FIGURE 2.

in magnitude, but is directed to the right of the line of flight.

The foregoing case is a special one, but from it may be discovered the Coriolis acceleration affecting an aircraft flying in a great circle at any point on the Earth, and in any direction. In Fig. 2, P_n and P_s are, as before, the Earth's poles of rotation; the great circle P_nPP_s is the meridian of the point P, which may be any point on the Earth. Pass a great circle through P in the direction of the aircraft's flight. The aircraft is moving with the velocity v. Now, if the Earth were rotating about P (instead of P_n) as a pole, we would have the case already discussed, that of a velocity along a meridian, and, from that case, calling ω' the angular velocity about P, we may write at once, $a = 2\omega' v \sin\phi'$, where ϕ' is the fictitious latitude of the point P, and ω' is that part of the Earth's angular velocity ω, about the pole P_n; that is the corresponding angular velocity about the pole P. The

*Mathematically equipped readers will readily paraphrase the discussion in terms of the calculus.

fictitious latitude is 90° for the case considered; hence $a=2\omega' v$. It remains to evaluate ω' in terms of ω.

From Fig. 2 it is seen that the angular distance between the poles P_n and P is $90°-\phi$, where ϕ is the true latitude of P. Turning now to Fig. 3, which is in the plane P_nPP_s, it is found that the angular velocity ω about P_n may be resolved into two components, ω', about P, and another component about a pole 90° from P. The second component is of no interest because it acts vertically, and hence plays no part in the magnitude or direction of the Coriolis acceleration. Laying off the vectors* representing the angular velocities in Fig. 3, it is seen at once from the figure that $\omega'=\omega \sin\phi$. Substituting this value in $a=2\omega' v$, we have $a=2\omega v \sin\phi$. Since the original direction of flight was not specified, the Coriolis acceleration is thus shown to be the *same*, whatever be the direction of motion. The numerical value depends only upon ϕ and v. (Obviously, the Coriolis acceleration is zero at the equator.)

In an infinitesimal time interval dt, the foregoing relation will cause a moving body, subject to *no* horizontal acceleration, to change its direction of motion

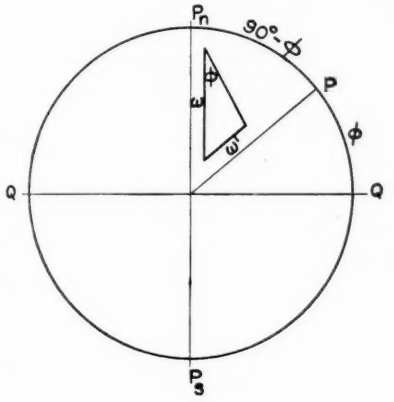

FIGURE 3.

relative to the Earth by the angle $d\theta$, where θ is also infinitesimal. This angle may be evaluated by the vector triangle of displacements in unit time, shown in Fig. 4. v is the longer leg and $a/2$, or $v\omega \sin\phi$, the shorter. From the figure, $\tan d\theta=\omega \sin\phi$. Now, ω is the measure of the rate at which the Earth is turning. Since this rate is, by the adoption of the Earth's rotation as the basis of time, the same as the rate at which time is elapsing, we may write $\omega=dt$; also since $d\theta$ is very small, $\tan d\theta=d\theta$. Substituting these values gives $d\theta=dt \sin\phi$. In a finite time, $\theta=t \sin\phi$.

This equation is that describing the turning of the vibration plane of the Foucault pendulum. Two things are notable about it; first, the motion or velocity is reciprocating, and second, rather remarkably, the rate of turning is independent of the velocity with which the pendulum bob is moving, and hence, is independent of the length of the pendulum and the amplitude of its swing.

It is clear that the Coriolis effect is distinct from those accelerations in the plane of the horizon, acting upon an aircraft, that are independent of the rotation of the Earth, and that such accelerations will be added algebraically to the Coriolis acceleration. As an

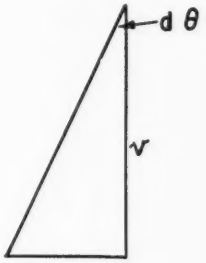

FIGURE 4.

$\frac{a}{2}=v\,\omega \sin\phi$

example, imagine an aircraft flying along a parallel of latitude of the Earth. (For clearness, imagine the aeroplane circling the North Pole at a short distance.) It is obvious that the aircraft must be continually "banked" to follow the parallel, and that for the acceleration involved, it is immaterial whether the Earth is rotating or not. The magnitude of this centripetal acceleration is that part of the total centripetal acceleration that lies in the horizon. A vector diagram will show that this horizontal acceleration is $a_T=v^2 \tan\phi/R$. For a speed of 600 m.p.h., in latitude 60°, this acceleration is 58 per cent as great as the corresponding Coriolis acceleration†. It becomes equal to the Coriolis acceleration, at this speed, at a latitude of about 74°.

It is evident that, in the future of high speeds flown in high latitudes, it will be necessary to include the effect of centripetal acceleration in all celestial altitudes measured in planes moving, during the observation, on rhumb lines. For more usual latitudes and more ordinary speeds, this centripetal acceleration is negligible. It is evident that it does not apply to guided missiles, which, once released, would follow great circles on a stationary Earth, and are, therefore, subject only to the Coriolis deflection on a moving Earth.

Deflections of path due to rotation of the Earth are of great importance, not only in celestial navigation and in gunnery, but in the paths taken by moving masses of air and bodies of water; that is, in the sciences of meteorology and oceanography. The effects on moving masses of air and water have, in their turn, climatic and biological consequences. For example, a barometric

*The convention of representation of these angular velocities as vectors is that the vector is laid parallel to the axis of rotation with the arrow in the direction of advance of an analogous right-handed screw turning in the same sense as the angular velocity.

†If the course is not due east or due west, then $a_T=(v^2/R)\tan\phi \sin C$ where C is the angle less than 90° made by the rhumb line and the meridian.

"low" in the atmosphere is an area where the atmospheric pressure is less than it is in the surrounding area. On a stationary Earth, air would flow in toward the "low" on radial lines, to equalise the pressure. On a rotating Earth, every part of the moving air mass is deflected from the radius; to the right, in the northern hemisphere; to the left, in the southern. The results are those large and small cyclonic, or rotating storms, so typical of the temperate zones. Similar deflections cause the N.E. and S.E. trade winds which, by blowing towards the equator, forever attempt to fill the depression of the "doldrums" or equatorial belt.

A Method of Calculating Stresses in a Non-Uniformly Thick Disc Subjected to Asymmetric Loads, Adapted to a Tabular Computation

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(Armstrong Siddeley Motors)

SYMMETRICALLY loaded circular plates of constant thickness are dealt with adequately in several of the standard textbooks on the strength of materials. The problem of non-uniformly thick plates subjected to symmetric loading has been discussed by M. Donath[1] and, more recently, by D. C. Boston[2]. So far very little has been done on the problem of a circular plate of varying thickness subjected to an asymmetric load. A tabular method approximating to the solution is outlined in this note. A typical example of the problem arises in certain manoeuvres of a jet aircraft in flight, the rotor discs of the engine being then subject to gyroscopic moments.

The method described covers the case of a circular disc of varying thickness, simply supported at its outer edge. It is subjected to a couple M acting in a plane, perpendicular to the disc, passing through the centre. The same method may be applied equally successfully to other boundary conditions. The method approximates to the disc shape by dividing it into a series of concentric uniformly thick annuli. Boundary conditions at the junction of adjacent annuli are equated and conditions in each annulus are obtained in terms of the value of the bending moments at the outer edge. The value of these is obtained from conditions at the inner edge.

NOTATION

w	deflection of plate (in.)
Q_x	shear force along a radial section, perpendicular to the section (lb./in.)
E_s	flexural rigidity of an annulus $= \dfrac{Eh^3}{12(1-\nu^2)}$
E	Young's modulus (lb./in.^{2})
h	thickness of plate (in.)
ν	Poisson's ratio
x	radial dimension (in.)
y	angular dimensions (radians)
M_x	radial bending moment (lb. in./in.)
M_t	tangential bending moment (lb. in./in.)
M_{xt}	torsional moment (lb. in./in.)
a	outer radius of plate (in.)
$Z = x/a$	
$S = M_x + M_t$	
M	applied bending couple (lb. in.)
W	value of S at $Z = 1$
A, B, C, D	constants in equation (1)

Received 31st July 1953.

1. METHOD

For a disc of uniform thickness (Fig. 1) subjected to an asymmetric load in the form of a couple, Timoshenko[3] gives the formula for deflections

$$w = (AZ + BZ^3 + CZ^{-1} + DZ \log_e Z) \cos y. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The origin of co-ordinates is taken at the centre of the disc, and y is measured from a diameter in the plane of action of the couple.

FIGURE 1.

It is assumed that a formula of this type holds good for each of the annuli of constant thickness into which the plate has been divided.

The following formulae are well known.

$$M_x = -E_s\left\{\frac{\partial^2 w}{\partial x^2} + \nu\left(\frac{1}{x}\frac{\partial w}{\partial x} + \frac{1}{x^2}\frac{\partial^2 w}{\partial y^2}\right)\right\}$$

$$M_t = -E_s\left\{\nu\frac{\partial^2 w}{\partial x^2} + \frac{1}{x}\frac{\partial w}{\partial x} + \frac{1}{x^2}\frac{\partial^2 w}{\partial y^2}\right\}$$

$$M_{xt} = E_s(1-\nu)\left(\frac{1}{x}\frac{\partial^2 w}{\partial x \partial y} - \frac{1}{x^2}\frac{\partial w}{\partial y}\right)$$

$$\Delta w = \frac{\partial^2 w}{\partial x^2} + \frac{1}{x}\frac{\partial w}{\partial x} + \frac{1}{x^2}\frac{\partial^2 w}{\partial y^2}.$$

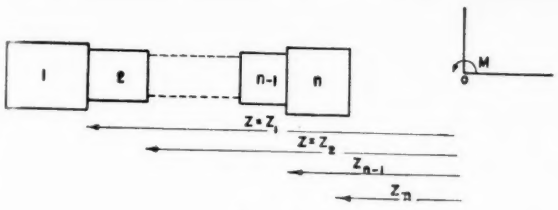

FIGURE 2.

Shearing force is given by

$$Q_r = -E_s \frac{\partial}{\partial x}(\Delta w).$$

Substituting for w in these expressions and considering conditions along the line of action of the couple, $y=0$.

$$M_r = -\frac{E_s}{a^2}\{BZ(6+2\nu)+CZ^{-3}(2-2\nu)+DZ^{-1}(1+\nu)\}$$

$$M_t = -\frac{E_s}{a^2}\{BZ(2+6\nu)-CZ^{-3}(2-2\nu)+DZ^{-1}(1+\nu)\}$$

$$Q_r = -\frac{E_s}{a^3}(8B-2DZ^{-2}).$$

Consider the disc split into a series of uniformly thick annuli, numbered 1 to n (Fig. 2).

For the disc simply-supported at its outer edge the following boundary conditions hold

$$(w)_{Z=1}=0, \qquad (M_r)_{Z=1}=0,$$

$$-a\int_{-\pi}^{\pi}(M_{rr})_{Z=1}\sin y\,dy + a^2\int_{-\pi}^{\pi}(Q_r)_{Z=1}\cos y\,dy + M = 0.$$

The method can be similarly applied to the condition of a clamped outer edge.

The last equation states the condition of equilibrium of the forces and moments acting at the boundary of the plate, and the external moment M.

The boundary conditions give

$$A+B+C=0 \quad . \quad . \quad . \quad (2)$$

$$B(6+2\nu)+C(2-2\nu)+D(\nu-3)=\frac{aM}{\pi E_s} \quad . \quad (3)$$

$$B(6+2\nu)+C(2-2\nu)+D(1+\nu)=0. \quad . \quad (4)$$

Let the value of S at $Z=1$ be W, then

$$W=\frac{-E_s}{a^2}(1+\nu)(8B+2D). \quad . \quad . \quad (5)$$

These equations refer to the outermost annulus only. Denote the coefficients of each section A_s, B_s, C_s, D_s ($s=1, \ldots\ldots, n$).

Equations (2), (3), (4), (5) lead to

$$A_1=\frac{-Wa^2}{4E_1(1-\nu)}, \quad B_1=\frac{-Wa^2}{8E_1(1+\nu)}+\frac{Ma}{16\pi E_1},$$

$$C_1=\frac{Wa^2(3+\nu)}{8E_1(1-\nu^2)}-\frac{Ma}{16\pi E_1}, \quad D_1=\frac{-aM}{4\pi E_1}.$$

Consider the junction between sections s and $s+1$. It is assumed that

(*i*) the deflection w is continuous at a junction.
(*ii*) the slope in the radial direction is continuous at a junction.
(*iii*) the radial bending moment is continuous at a junction.
(*iv*) the radial shearing force is continuous at a junction.

Hence

$$A_sZ_s+B_sZ_s^3+C_sZ_s^{-1}+D_sZ_s\log_e Z_s = A_{s+1}Z_s+B_{s+1}Z_s^3+C_{s+1}Z_s^{-1}+D_{s+1}Z_s\log_e Z_s,$$

$$A_s+3B_sZ_s^2-C_sZ_s^{-2}+D_s(1+\log_e Z_s) = A_{s+1}+3B_{s+1}Z_s^2-C_{s+1}Z_s^{-2}+D_{s+1}(1+\log_e Z_s),$$

$$E_s\{B_sZ_s(6+2\nu)+C_sZ_s^{-3}(2-2\nu)+D_sZ_s^{-1}(1+\nu)\} = E_{s+1}\{B_{s+1}Z_s(6+2\nu)+C_{s+1}Z_s^{-3}(2-2\nu)+D\,Z_s^{-1}(1+\nu)\},$$

$$E_s(4B_s-D_sZ_s^{-2})=E_{s+1}(4B_{s+1}-D_sZ_s^{-2}).$$

For $\nu=\frac{1}{3}$,

$$B_{s+1}=B_s\left(\frac{11}{12}\frac{E_s}{E_{s+1}}+\frac{1}{12}\right)+C_sZ_s^{-4}\left(\frac{1}{12}\frac{E_s}{E_{s+1}}-\frac{1}{12}\right)+ +D_sZ_s^{-2}\left(\frac{-1}{24}\frac{E_s}{E_{s+1}}+\frac{1}{24}\right)$$

$$C_{s+1}=B_sZ_s^4\left(\frac{3}{4}\frac{E_s}{E_{s+1}}-\frac{3}{4}\right)+C_s\left(\frac{1}{4}\frac{E_s}{E_{s+1}}+\frac{3}{4}\right)+ +D_sZ_s^2\left(\frac{3}{8}\frac{E_s}{E_{s+1}}-\frac{3}{8}\right)$$

$$D_{s+1}=B_sZ_s^2\left(\frac{-1}{3}\frac{E_s}{E_{s+1}}+\frac{1}{3}\right)+C_sZ_s\left(\frac{1}{3}\frac{E_s}{E_{s+1}}-\frac{1}{3}\right)+ +D_s\left(\frac{5}{6}\frac{E_s}{E_{s+1}}+\frac{1}{6}\right).$$

A_1, B_1, C_1, D_1 are known in terms of W, hence a table (Table I) can be constructed to give A_s, B_s, C_s, D_s in terms of W.

For a disc clamped to a hub at its inner edge

$$\left(\frac{\partial w}{\partial Z}\right)_{Z=Z_n}=\left(\frac{w}{Z}\right)_{Z=Z_n}$$

leading to

$$2B_nZ_n^2-2C_nZ_n^{-2}+D_n=0,$$

which is a linear equation in W.

For a disc simply-supported at its inner edge, assuming a rigid hub,

$$(M_r)_{Z=Z_n}=0$$

i.e.

$$5B_nZ_n+C_nZ_n^{-3}+D_nZ_n^{-1}=0.$$

B_s, C_s, D_s are now known, on substitution for W, and bending moments are obtained from

$$(M_r)_s=\frac{-E_s}{a^2}\left(\frac{20}{3}B_sZ+\frac{4}{3}C_sZ^{-3}+\frac{4}{3}D_sZ^{-1}\right)$$

$$(M_t)_s=\frac{-E_s}{a^2}\left(4B_sZ-\frac{4}{3}C_sZ^{-3}+\frac{4}{3}D_sZ^{-1}\right).$$

The radial and hoop stresses are given respectively by

$$\sigma_r=\frac{6(M_r)_s}{h_s^2} \qquad \sigma_t=\frac{6(M_t)_s}{h_s^2}.$$

TABLE I

STRESSING OF DISC SUBJECT TO GYROSCOPIC COUPLE

$\frac{E_s}{E_{s+1}}=$			$Z_s^2=$	$Z_s^4=$			$B_s=$	$C_s=$			$D_s=$		
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)	(m)	
$\frac{11}{12}\frac{E_s}{E_{s+1}}$	(a)$+\frac{1}{12}$		(b)$\times B_s$	$\frac{1}{12}\frac{E_s}{E_{s+1}}$	(e)$-\frac{1}{12}$	(f)$/Z_s^4$	(g)$\times C_s$	$\frac{-1}{24}\frac{E_s}{E_{s+1}}$	(i)$+\frac{1}{24}$	(j)$/Z_s^2$	(k)$\times D_s$	(d)$+$(h)$+$(l)	
													B_{s+1}
(n)	(o)	(p)	(q)	(r)	(s)	(t)	(u)	(v)	(w)	(x)	(y)	(z)	
$\frac{3}{4}\frac{E_s}{E_{s+1}}$	(n)$-\frac{3}{4}$	(o)$\times Z_s^4$	(p)$\times B_s$	$\frac{1}{4}\frac{E_s}{E_{s+1}}$	(r)$+\frac{3}{4}$		(s)$\times C_s$	$\frac{3}{8}\frac{E_s}{E_{s+1}}$	(v)$-\frac{3}{8}$	(w)$\times Z_s^2$	(x)$\times D_s$	(q)$+$(u)$+$(y)	
													C_{s+1}
(a′)	(b′)	(c′)	(d′)	(e′)	(f′)	(g′)	(h′)	(i′)	(j′)	(k′)	(l′)	(m′)	
$-\frac{1}{3}\frac{E_s}{E_{s+1}}$	(a′)$+\frac{1}{3}$	(b′)$\times Z_s^2$	(c′)$\times B_s$	$\frac{1}{3}\frac{E_s}{E_{s+1}}$	(e′)$-\frac{1}{3}$	(f′)$/Z_s^2$	(g′)$\times C_s$	$\frac{5}{6}\frac{E_s}{E_{s+1}}$	(i′)$+\frac{1}{6}$		(j′)$\times D_s$	(d′)$+$(h′)$+$(l′)	
													D_{s+1}

2. SUMMARY OF METHOD

1. Divide the disc into n annuli of constant thickness.
2. Calculate E_s for each annulus.
3. Calculate $\frac{E_s}{E_{s+1}}$, $\frac{6}{h_s^2}$, Z_s^2, Z_s^4.
4. Calculate B_1, C_1, D_1 in terms of W.
5. Calculate B_s, C_s, D_s from Table I ($s=2\ldots\ldots$) in terms of W.
6. Use the boundary conditions at the inner edge of the disc to obtain W.
7. Substitute for W in expressions for B_s, C_s, D_s and calculate bending moments in each section.
8. Calculate stresses from bending moments.

3. CONCLUSION

At the time the theory was completed some test results were received which could be compared with the theory. The test was not designed to test out the theory and any comparison of figures bore out this fact. In the test a disc was subjected to an axial load and a couple, to simulate conditions on a rotor disc when a jet aircraft turns in flight. The disc was simply supported over most of the circumference but a small portion was clamped. The inner hub was fixed by a Hirth coupling and could be regarded as something between a clamped and a simply-supported hub.

Theoretical figures were calculated for a simply-supported outer hub combined with a clamped inner hub, and a simply-supported inner hub. These showed a fair agreement with the experimental figures as recorded by strain gauges along the disc. It can reasonably be deduced that, had the experiment been designed specifically to prove the theory and had correct boundary conditions been simulated, the experimental and theoretical figures would have been closely in agreement.

ACKNOWLEDGMENT

The author expresses his thanks to Mr. D. R. Evans, Head of Stress Department, and to Armstrong Siddeley Motors Ltd., for permission to publish this work.

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THE LIBRARY
Reviews

FLAMES. THEIR STRUCTURE, RADIATION AND TEMPERATURE. *A. G. Gaydon and H. G. Wolfhard. Chapman & Hall, London 1953. 340 pp. Diagrams and photographs. 55s. net.*

It is pleasing to welcome a British book on this subject and particularly one written by the present authors who are responsible for many outstanding contributions to the literature and are particularly well qualified to survey the mass of published work which has appeared in recent years.

The aim of the authors has been to give a fairly advanced discussion of a part of the field, namely that concerned with stationary flames, with the emphasis on the physical rather than the chemical viewpoint. In this they have succeeded admirably, but it must be made clear that the greater part of the book deals with problems mainly of scientific interest and no attempt is made to deal with topics of essentially technical importance. Subjects which have received particular attention are the measurement of burning velocities, theories of flame propagation, the formation of carbon in flames, flame radiation, the measurement of flame temperatures and ionisation in flames. The chapters on burning velocity measurement and on theories of flame propagation give clear and valuable summaries of the present state of knowledge and in connection with flame propagation, the reviewer is glad to see that the authors, in the words of their preface . . . " have avoided purely mathematical discussions " . . . " mathematical treatments of combustion " . . . " usually involving many unknown and often unknowable parameters."

An interesting section of the book deals with diffusion flames, with particular reference to spectroscopic studies of flat diffusion flames. This, to some extent, augments information presented in Gaydon's *Spectroscopy and Combustion Theory*.

Two chapters deal with the measurement of flame temperatures, attention being paid both to the well known spectrum line reversal method and to other methods which have been employed. A valuable chapter deals with the calculation of flame temperatures and the composition of the flame gases. In this, the useful method of Damköhler and Edse is described in some detail.

References are given according to the Harvard system (which in the opinion of the reviewer is more suitable for use in short papers than in a book). Author and subject indexes are provided.

The subject matter of this stimulating book represents a field in which much research is being done, in the words of the authors it—" is in the flame front of combustion research." This means that parts of the book will perhaps become out of date fairly rapidly. It provides, however, a clear review of many controversial points.—R. LONG (Birmingham University).

DIRECT CURRENT MACHINES FOR CONTROL SYSTEMS. *A. Tustin. 306 pp. Diagrams. E. & F. Spon Ltd., London 1952. 50s. net.*

The development of the theory of automatic control mechanisms over the past fifteen years has to a large extent been in the hands of electronic engineers in the communication and " radar " fields of activity. The subject thus contains very largely the terminology and techniques of the " light current " engineer, but the author, who is eminent in the automatic control field, refrains from using any terms or introducing any analytical methods foreign to the " power " engineer in the field of rotating electrical machinery. The book begins with a chapter, a fifth of the whole book in length, on the theory of control systems and their analysis. It shows the reason why a knowledge of the transient or dynamic characteristics of machines is so essential.

The rest of the book deals with direct current machinery principally, separately excited machines, compound machines and combinations of these two; the Ward-Leonard arrangement, the Rosenberg dynamo, the Metadyne, Amplidyne, Magnicon, Magnavolt and Rototrol. A unified theory is built up and runs through the book clarifying the similarities and essential differences between the various machines. A final chapter deals with limited movement (rotary and linear motions) commutator-less D.C. machines.

The book meets the needs of the control engineer not familiar with machine analysis and the machines engineer requiring knowledge of the application and development of machinery in control systems. It is excellent in that it uses only elementary mathematics to the standard of simple linear differential equations and assumes only a basic minimum of machine theory. It is probably the only book devoted solely to this subject.

The author makes much use of his " scheme of dependencies " whereas many readers would prefer a more straightforward schematic block diagram. The analysis on page 114 of the effect of a damping winding is misleading, for in many cases the result would be to increase the value of one of the time constants already in existence instead of introducing a further time constant.

The book will be of use to anyone interested in automatic control.—JOHN C. WEST (The University, Manchester).

LUBRICANTS AND LUBRICATION. *Selected Government Research Reports. Volume 11. H.M.S.O. London 1952. 234 pp. 35s. net.*

This volume includes nineteen research reports and seven short abstracts of relevant reports. The main reports are dated from 1940 to 1949 and, as the date of publication of the bound volume is 1952 and it has only recently become available, several of the reports are now out of date with regard to physical information. The methods employed, however, could be used again with more modern materials where up to date information is essential.

The work covers a wide range of materials and conditions—from aeration of lubricating oils in piston engines to boundary friction in silicone fluids and greases, including lubrication of clockwork mechanisms and oils and greases for the treatment of leather.

Much painstaking work is covered but a few criticisms can be made. The first is, why were not a number of these reports published in this handy form several years ago instead of waiting until they have become out of date? Also with the first, and oldest, it would have been most helpful if a rider had been added stating whether the device recommended had been satisfactory in service conditions.

In the second report, now ten years old, the results on low temperature grease are now quite out of date. The methods of test, however, are still quite useful.

The third report deals with low temperature viscosities of diluted lubricating oil to Specification D.T.D.472B, a secret specification which has long been superseded by D.T.D.2472B.

In the fifth, torques are quoted in gm./cm. instead of gm.cm.; also much time in reading is wasted because of inadequate cross referencing between tables.

The addition of oil to kerosine for gas turbines is now forbidden but the method of determining the oil content might be used for determining contamination in test rigs, which is a very real problem.

The physical properties of D.E.D.2472/B.O. are most useful, there are numerous curves and eight tables, and the report No. 9 gives some useful practical results obtained at the low temperature research station, Fort Churchill.

The tenth report covers a subject which certainly needs cleaning up—consistency of soft solids and thickened liquids; the next is a sequel dealing with the plate viscometer in greases, cement, clay suspensions, etc.

No. 15 covers greases for optical instruments including D.T.D.143C and D.T.D.577, both of which are now obsolete.

The last three reports cover the boundary friction of silicone fluids and greases and the academic apparatus used to measure it. There has been considerable development in silicone products since 1947 and this wants bringing right up to date. The preparation of metallic surfaces for test is dealt with in considerable detail and should be extremely useful.

Generally, at 35/- this book cannot be ignored by anyone interested in lubrication.—A. E. BINGHAM.

THE AIRCRAFT COMMANDER IN COMMERCIAL AIR TRANSPORTATION. *Dr. M. S. Kamminga. Martinus Nijhoff, The Hague, 1953. 184 pp. 10 guilders net.*

The title does not reveal that this book is primarily a study of the legal position of the Aircraft Commander in international air transport. Although printed and produced in The Netherlands, Dr. Kamminga has written in English, and thereby made the results of his studies readily available to the international aviation world. The subject matter is treated under two general headings. Part I deals with the Aircraft Commander in Dutch Law: and Part II discusses the current (1947) I.C.A.O. draft Convention on the Legal Status of the Aircraft Commander, which is itself reprinted as an Appendix to the volume.

Although Part I is mainly concerned with Dutch Law, the business of civil aviation is far too international to allow an adequate treatment inside the legal framework of a single nation, and Dr. Kamminga makes many references and excursions outside his self-imposed national boundary. To this extent Part I comprises a typical study of the unsatisfactory position of the Aircraft Commander as expressed in current national laws; it therefore provides an excellent starting point from which to begin an examination of the requirements of a generally acceptable legal status for the Aircraft Commander.

In Part II Dr. Kamminga proceeds to outline the history behind the current I.C.A.O. draft Convention, and the extensive groundwork carried out by C.I.T.E.J.A. (Comité International Technique d'Experts Juridiques Aériens) starting in 1926, and finally handed over to the Legal Committee of I.C.A.O. in 1947. For over a quarter of a century successive international committees have studied the problems involved, and, while civil aviation has developed from insignificant beginnings into a great commercial industry, no practical steps have been taken to define the rights and duties of the Aircraft Commander in international law.

Obviously agreement on such a matter can only be reached after detailed technical study by legal experts, and although Dr. Kamminga makes some proposals for the future, his main purpose has been to present the position as it is to-day. He has rendered a valuable service in bringing together the relevant facts, the mere recital of which is sufficient to indicate how unsatisfactory is the present position. Not only may appropriate legal action be withheld owing to doubts of the competency of a national court to give judgment on events occurring over another State, or over the high seas, but also the personal rights, duties, and liabilities, of the individual aircraft commander remain indeterminate, and his very existence lacks effective legal recognition. As far as liability is concerned in the case of accident, Dr. Kamminga has some pertinent comments on the over-worked phrase "pilot error," so beloved of courts of inquiry. The need for a clear definition of this phrase cannot be too strongly emphasised.

The Aircraft Commander has "an important and responsible task to perform in commercial air transportation." A suitable Convention granting him legal status appropriate to his peculiar and unique position is certainly long overdue. This volume, which is surprisingly readable, is recommended as a useful study for readers interested either in Air Law, or in Airline Pilotage. The main criticism from a British point of view must be the very limited number of references to Civil Air Legislation as at present in force in the United Kingdom. In conclusion one must express the hope that Dr. Kamminga's book will itself, by giving a wider publicity to the issues involved, assist in the hastening of effective international action in this much neglected field.—A. M. A. MAJENDIE.

ACROSS THE PARALLEL. *George Odgers. Heinemann 1953. 239 pp. Map and Illustrations. 21s. net.*

A professional journalist, George Odgers re-enlisted in the R.A.A.F. on the outbreak of the Korean disturbance and was posted to No. 77 Fighter Squadron, the first British Commonwealth unit in this theatre to take off for battle.

His book tells of what he saw and heard of this Communist-inspired civil war, brother against brother—the most pitiful and savage of human conflicts. Yet it was more than that, for, at the intervention of the U.S.A., the fifty-eight signatories of the United Nations were on trial in the terms of their charter: "Determined to save succeeding generations from the scourge of war . . . to unite our strength to maintain peace and security." As symbol of this endeavour the author takes 77 Squadron and weaves an extensive narrative of the first year of the Korean war around its small group of pilots, setting their individual experiences as a foil to the general picture of the war.

Nevertheless, little of the atmosphere of flying emerges, although the portrayal of events from the focal point of air operations is clear enough. The crux of the situation is never revealed, nor is there sense of events even though they are chronicled. Indeed, only by viewing a war from beginning to end, and then at some distance, could history be portrayed without feeling of confusion. Anything else can attempt little more than a vignette of atmosphere, or the detailed portrayal of the cause and effect of a sequence of events leading to a major situation, but what George Odgers has done is write the Squadron history of a brief period.—H. J. PENROSE.

ELECTRIC RESISTANCE HEATING. *British Electrical Development Association. 182 pp. Diagrams, tables and photographs. 8s. 6d. net.*

Number 5 in the Electricity and Productivity Series, produced by the B.E.D.A. This book gives examples of the application of Electric Resistance Heating to equipment ranging from large heat treatment furnaces to soldering irons. It is claimed that it is the first book to be published on this particular branch of electric heating and as such it should be of great value to production engineers.

PRESENTATION OF NUMERICAL VALUES. *B.S. 1957. British Standards Institution 1953. 15 pp. 3s. net.*

This new British Standard, prepared in conjunction with about thirty professional Institutions, contains in ten pages Sections on:

Kinds of numerical values considered.

Meaning of the terms "Number of significant figures" and "Number of decimal places."

Number of significant figures to retain.

Rounding of numbers.

Layout of numbers.

Anyone working with figures would be well advised to have this Standard by him and to accustom himself to its recommendations. It has the added merit of being clearly written in the most concise and direct English.

HAZARDOUS INDUSTRIES. *Edited by A. S. Minton and W. Thomas. Chantry Publications, for the Institution of Fire Engineers 1953. 106 pp. 6s. net.*

This small book is concerned with ways of preventing and dealing with fires in various industries in which fires are likely to occur. There is a small section on Aircraft Factories.

TWO DECADES OF HELICOPTER DESIGN. *Temple Press 1953. 14 pp. Diagrams and photographs. 3s. 6d. net.*

This booklet is a reprint of six tables, which were originally printed in *The Aeroplane*, giving main particulars of rotating wing aircraft built in the past twenty years. The tables are accompanied by photographs and diagrams and a short historical survey by L. H. Hayward.

HOW PLANES FLY. *S. E. Veale and "Wren." Puffin Picture Book 94. Penguin Books Ltd. 1953. 31 pp. Illustrated. 2s. 6d. net.*

Harassed fathers who try to adapt their technical minds to their small sons' inquiries on the mysteries of flight can now have all the work done for them for a modest half-crown. Mr. Veale provides a lucid text (without "talking down") and Wren makes it even easier to understand with clear illustrations. Christmas is coming.

THE FLYING OMNIBUS. *Edited by Paul Jensen. Cassell 1953. 15s. net.*

The title of this book is confusing. It does not refer to a development in helicopters but is an omnibus collection of specimens of aeronautical literature.

The book is divided into five "parts" chronologically and ranges from a hoax perpetrated by Edgar Allen Poe (recording an Atlantic crossing by balloon) to a fantasy in the Dan Dare idiom. Some items are fiction and some are fact and the authors range from James Thurber to Anne Morrow Lindbergh. Something for everyone at all ages and all intellectual levels.

THE SEA SHALL NOT HAVE THEM. *John Harris. Hurst & Blackett. London 1953. 256 pp. 9s. 6d. net.*

The story of the search for four survivors of a Hudson in the North Sea in 1944. A well-told story of four men in a rubber dinghy; of the crew of a high speed launch of Air Sea Rescue; of those who organise the search from the ground, and in the air. Although classed as fiction it might well be fact for the author knows the sea, knows the Air Force and served for three years with the Air Sea Rescue Service.

Additions to the Library

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Bowden, B.V. (*Editor*). FASTER THAN THOUGHT. Pitman. 1953.
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Reports

AERODYNAMICS

Compressible Flow

On reflection of shock waves from boundary layers. H. W. Liepmann, A. Roshko and S. Dhawan. N.A.C.A. Report 1100 (1952).

Measurements are presented at Mach numbers from about 1·3 to 1·5 of reflection characteristics and the relative upstream influence of shock waves impinging on a flat surface with both laminar and turbulent boundary layers. The difference between impulse and step waves is discussed and their interaction with the boundary layer is compared. General considerations on the experimental production of shock waves from wedges and cones and examples of reflection of shock waves from supersonic shear layers are also presented.—(1.2.3 × 1.1).

Fluid Dynamics

Impingement of water droplets on wedges and diamond airfoils at supersonic speeds. J. S. Serafini. N.A.C.A. Technical Note 2971 (*July* 1953).

An analytical solution has been obtained for the equations of motion of water droplets impinging on a wedge in a two-dimensional supersonic flow field with a shock wave attached to the wedge. The analytical expressions are utilised to determine the impingement on the forward surfaces of diamond aerofoils in supersonic flow fields with attached shock waves.—(1.4).

Internal Flow

Some tests on compressor cascades of related aerofoil having different positions of maximum camber. A. D. S. Carter. R. & M. 2694 (*December* 1948, *published* 1953).

One of the major variables defining the shape of any blade is its position of maximum camber, and there are several indications that its choice considerably effects the performance of the cascade. Tests have therefore been made on a series of aerodynamically equivalent cascades in which the position of maximum camber was varied systematically. The tests covered a full incidence range up to choking.—(1.5.4).

The structure of turbulence in fully developed pipe flow. J. Laufer. N.A.C.A. Technical Note 2954 (*June* 1953).

Measurements, principally with a hot-wire anemameter, were made in fully developed turbulent flow in a 10-inch pipe at speeds of 10 and 100 feet per second.—(1.5.1).

Loads

See also Structures, Loads

The measurement of the overall drag of an aircraft at high Mach numbers. D. J. Higton et. al. R. & M. 2748 (*January* 1949, *published* 1953).

This report describes the technique which has been developed to measure the overall drag of an aircraft at high Mach numbers in both level flight and dives. It shows how improvements have been made both in flight and tunnel numbers in both level flight and dives. It shows how improvements have been made both in flight and tunnel technique so that comparisons between full-scale and model tests have now become possible. Flight results from Meteor IV aircraft show close agreement between drag measured in level flight and in dives and later tests compare well with high-speed wind-tunnel measurements on a 1/12th scale model.—(1.6).

Drag of circular cylinders for a wide range of Reynolds numbers and Mach numbers. F. E. Gowen and E. W. Perkins. N.A.C.A. Technical Note 2960 (*June* 1953).

Pressure distribution measurements on a two-dimensional circular cylinder have been made at high subsonic and supersonic speeds. Drag coefficients obtained from these measurements are presented with results from other sources. No effects of Reynolds number were found at supercritical Mach numbers. Effects of fineness ratio on drag of three-dimensional cylinders at supersonic speeds were investigated and found to be small.—(1.6).

Stability and Control

The concept of longitudinal static stability. J. J. Thompson. R.A.A.F. Tech. Note Aero. 9 (*July* 1952).

In Part 1, the term "static stability" is discussed, and the conclusion reached that its meaning is given only by its place in the mathematical theory of stability, assuming small displacements from an equilibrium state. Only in particular cases is it possible to give an adequate physical description and relate it to the feel of the aircraft. In general, static stability depends on the constant term in the stability polynomial, and this term also arises from the equations governing the steady state control characteristics. Part 2 is therefore concerned with formulating, in terms of the general theory of aircraft stability, the relations between elevator angle, tab angle, C.G. position, forward speed etc., as a basis for the measurement of static stability by flight testing.—(1.8.2).

Longitudinal stability and control—quantitative requirements and tests. J. J. Thompson. R.A.A.F. Tech. Note Aero. 10 (*September* 1952).

A comparison has been made of quantitative requirements and tests for longitudinal stability and control, as laid down in A.P.970 and the U.S.A.A.F. Specification R-1815-A over the speed range between the stall and the onset of compressibility effects. It is concluded that advantages are to be gained by incorporating ideas from both spheres in future flight testing in this subject.—(1.8.2).

Flight investigation of the effect of control centering springs on the apparent spiral stability of a personal-owner airplane. J. P. Campbell, et al. N.A.C.A. Report 1092 (1952).

A flight investigation has been conducted on a typical high-wing personal-owner aeroplane to determine the effect of control centring springs on apparent spiral stability. The centring springs were used in both the aileron and rudder control systems to provide both a positive centring action and a means of trimming the aeroplane.—(1.8.3).

Wings and Aerofoils

The effect of compressibility on the performance of a Griffith aerofoil. H. H. Pearcey and E. W. E. Rogers. R. & M. 2511 (*November* 1946, *published* 1953).

Experiments have been made in the 20 in. × 8 in. High Speed Tunnel at the National Physical Laboratory on a 9 in. chord, 22 per cent. thick, symmetrical Griffith section at 0 deg. incidence. Drag was determined by the pitot traverse method. Information on the flow was obtained from the pitot tube traverses, from direct shadow photographs, and from normal pressure measurements. Three Mach numbers of the undisturbed stream were covered, namely 0·4, 0·6 which is just below the theoretical critical, 0·65, for the section, and 0·7 at which shock waves were present. Estimates of the power absorbed by the compressor, ignoring duct losses, are made from measurements of the mass of air sucked and the static pressure in the slots. Additional information was obtained on the adverse effect of a large radius of the forward lip of the slot, on the effect at $M_0 = 0{\cdot}4$ of an increase in slot width, and on the choke quantities for the slots.—(1.10.2.1).

A comparison of two methods of calculating wing loading with allowance for compressibility. V. M. Falkner. R. & M. 2685 (*October* 1949, *published* 1953).

The report gives the results of a comparison by two different methods of the aerodynamic loading of a tapered V wing of aspect ratio 5·8 and 45 deg. sweepback at $M = 0{\cdot}8$.

NOTE:—*The figures in parenthesis at the end of each Summary are for office use only.*

based on the Prandtl-Glauert factor or linear perturbation theory: the first method, associated particularly with vortex-lattice theory, deals with changes in Mach number by preserving the plan of the wing and using special Tables of downwash, while the second uses the solution for Mach number 0 on a wing with the lateral dimensions reduced by a specified factor. Examples of the application of the theory are also given for a delta wing, for a straight tapered wing without sweep, and for a tapered wing with 28·4 deg. sweepback. It is possible to give a general and reasonable explanation of the nature of the variations of load grading and local aerodynamic centre which occur with increasing Mach number, and with the information given, there should be no difficulty in the prediction of Mach number effects on a wide range of plan forms.—(1.10.1.2).

The effect of leading edge wedges on the stalling behaviour of the Wirraway. F. W. Barnes and B. G. Newman. R.A.A.F. Tech. Note Aero. 8 (*November* 1951).

The wing drop, which the Wirraway exhibits at the stall, has been considerably reduced by fastening small wedges to the leading edge of the wing near the root. Measurements of the stalling speeds are presented together with a qualitative appraisal of the improvement. Some understanding of the action of the wedges has been obtained by photographing tufts.—(1.10.2.2).

Analysis of the effects of wing interference on the tail contributions to the rolling derivatives. W. H. Michael, Jr. N.A.C.A. Report 1086 (1952).

An analysis of the effects of wing interference on the tail contributions to the rolling stability derivatives of complete aeroplane configurations is made by calculating the angularity of the air stream at the vertical tail due to rolling and determining the resulting forces and moments. Some calculated sidewash results for a limited range of wing plan forms and vertical-tail sizes are presented. Equations taking into account the sidewash results are given for determining the tail contributions to the rolling derivatives.—(1.10.1.2).

Theoretical investigation of the supersonic lift and drag of thin, sweptback wings with increased sweep near the root. D. Cohen and M. D. Friedman. N.A.C.A. Technical Note 2959 (*June* 1953).

Formulae are derived by the use of linear theory for the lift and drag due to lift, at supersonic speeds, of thin flat wings having a discontinuity in the leading-edge sweep, with the inboard portion of the leading edge very highly swept, the outboard portion less so. Examples show the effect of the bend in the leading edge on the pressure distribution, the lift-curve slope, and drag. The results are also related to the effect of a wing-fuselage juncture on the lift on the wing.—(1.10.1.2).

Testing and Instruments

See also Research

The photographic determination of the relative position of aircraft. J. J. Thompson. R.A.A.F. Tech. Note Aero. 6 (*October* 1951).

To determine the vertical height of an aircraft relative to an airborne camera station, the equations relating its position and orientation in space with the position and geometry of its image on the film are derived. A few solutions are given for cases of practical importance, when the distance of the image from the lens is the focal length. These solutions are simplest if light rays from the aircraft to the optical centre of the lens can be assumed parallel.—(1.12).

Altimeter and A.S.I. pressure error corrections. J. J. Thompson. R.A.A.F. Tech. Note Aero. 7 (*October* 1951).

The necessary corrections to aircraft altimeter and air speed indicator readings because of pressure errors in the pitot and static lines are discussed, and charts, tables, and formulae included to facilitate their calculation. Particular attention is devoted to relating these errors to the flow at the pitot-static head and its location relative to the aircraft, in order to clarify the role of compressibility in the effect of varying altitude. As a general discussion of the problem only is attempted, no definite conclusions are reached concerning the correct form for the variation of the static pressure error coefficient with Mach number.—(1.12).

AIRCRAFT DESIGN

The elimination of unwanted light reflections from Mustang fuel gauge glasses. N. A. Plumb. R.A.A.F. Tech. Note Eng. 1 (*July* 1951).

A simple modification to Mustang fuel gauge bezels has been developed which effectively reflects unwanted light rays away from the pilot's line of sight. The prototype was effected on Mustang aircraft A68-143.—(4.2.1 × 18).

AIRCRAFT OPERATION

Evaporation of drops of liquid. J. K. Hardy. R. & M. 2805 (*March* 1947, *published* 1953).

An analysis has been made of the processes which follow when a drop of liquid is subjected to a sudden change in the condition of the air in which it is suspended. Equations are given from which either the temperature of the drop, or the rate at which it will evaporate, can be calculated.—(5.4.1).

Effect of ice and frost formations on drag of N.A.C.A. 65_1-212 *airfoil for various modes of thermal ice protection. V. H. Gray and U. H. von Glahn. N.A.C.A. Technical Note* 2962 (*June* 1953).

Studies were made to determine the effect of ice and frost formations on the drag of an 8-foot-chord N.A.C.A. 65_1-212 aerofoil. Frost formations on aerofoil surfaces caused large drag increases and may result in stalling of the aerofoil.—(5.4.1).

FLIGHT TESTING

The correction of level speeds at constant heights. F. J. P. Wood. R.A.A.F. Tech. Note Aero. 2 (*May* 1949).

The procedures of A.A.E.E. Report Res/170 and the addendum thereto, for the correction of level speeds at constant height, are recapitulated, and the effect is examined of using a modified law of engine power variation with temperature.—(13.1).

FUELS AND LUBRICANTS

Reaction processes leading to spontaneous ignition of hydrocarbons. C. E. Frank and A. U. Blackham. N.A.C.A. Technical Note 2958 (*June* 1953).

The present study has been initiated to uncover fundamental data on the oxidation intermediates leading to the spontaneous ignition of hydrocarbons. This investigation was conducted at the Applied Science Research Laboratory of the University of Cincinnati under the sponsorship and with the financial assistance of the National Advisory Committee for Aeronautics.—(14.1 × 32.1).

MATERIALS

A combined dilatometer-electrical resistivity apparatus for investigating the sintering of metal powder compacts. N. A. McKinnon. A.R.L. Australia. Report SM. 205 (*April* 1953).

An apparatus to measure the changes in linear dimensions and electrical resistivity of pressed compacts of metal powders during sintering is described. The usefulness of this apparatus has been shown by some investigations into the sintering of copper, aluminium and aluminium alloys.—(21.1 × 21.2.2).

The effects of stepped loadings in static reversed stress tests on annealed copper. R. B. Davies. A.R.L. Australia. Report SM. 203 (*March* 1953).

As an approach to the study of structural and dimensional changes occurring during the period of load build-up in direct-stress fatigue tests, stepped reversed-stress tests have been made under "static" test conditions on annealed copper test-specimens. The results of tests in which loading to reversed-stress ranges not exceeding ±7 tons/in.² was achieved either by simple reversal of stress or by one of various types of stepped loading, are reported. The significance of loading rates in direct-stress fatigue tests is discussed.—(21.2.2).

MATHEMATICS

Approximate solution of simultaneous equations by means of transformation of variables. Applications to aeronautical problems. B. Langefors. S.A.A.B. T.N. 7 (*October* 1952).

Matrix methods to establish s stems of equations are described. These methods are generalised to permit approximate reduction of big calculations by aids of routine matrix calculations. The method can be used to establish greatly reduced first approximations for pre-checking purpose as well as for the shortening of final calculations. Applications to statics, flutter and aerodynamics are indicated briefly.—(22.1).

POWER PLANTS

A survey of calibration equipment for engine speed indicators. K. J. Holmes. R.A.A.F. Tech. Note Inst. 1 (*June* 1951).

This report is the result of an investigation of the problem of calibrating Engine Speed Indicators used for performance testing in this Unit. A number of relevant pieces of apparatus are discussed and a table is presented to show the accuracy attainable by each.—(27.1 × 27.2.2 × 18.1).

Air admixture to exhaust jets. E. Sanger. N.A.C.A. Technical Memorandum 1357 (1950, *published July* 1953).

The problem of thrust increase by air admixture to exhaust jets of rockets, turbo-jet, ram- and pulse-jet engines is investigated theoretically. The optimum ratio of mixing chamber pressure to ambient pressure and speed range for thrust increase due to air admixture is determined for each type of jet engine.—(27).

Physical properties of concentrated nitric acid. W. L. Sibbitt, et al. N.A.C.A. Technical Note 2970 (*June* 1953).

From a review of the literature and additional experimental measurements, recommended values of the physical properties of white fuming nitric acid were determined for the temperature range of approximately −35° to 300° F. These properties included thermal conductivity, dynamic viscosity, specific heat, density, and vapour pressure. Values were determined within an accuracy of 5 per cent. for all physical properties except the vapour pressure.—(27.3 × 32.2).

PROPELLERS

Surveys of the flow fields at the propeller planes of six 40° *sweptback wing-fuselage-nacelle combinations. V. L. Rogallo and J. L. McCloud III. N.A.C.A. Technical Note* 2957 (*June* 1953).

The flow fields at the propeller planes of six 40° sweptback, semi-span wing-fuselage-nacelle combinations have been surveyed. The significance of the flow-field parameters in terms of propeller-oscillating aerodynamic loads is indicated. Comparisons of measured and predicted upflow angles are made for all six models.—(29.1).

Propeller-performance charts for transport airplanes. J. Gilman, Jr. N.A.C.A. Technical Note 2966 (*July* 1953).

The preliminary selection of a propeller on the basis of cruising and take-off performance for application to transport aeroplanes at flight Mach numbers up to 0·8 can be accomplished by the use of the charts and methods presented. The charts are of sufficient scope to permit a fairly rapid evaluation of the propeller performance for engine power ratings of 1,000 to 10,000 horsepower. The method is presented primarily in the interest of propeller-noise abatement.—(29.6).

Propeller-noise charts for transport airplanes. H. H. Hubbard. N.A.C.A. Technical Note 2968 (*June* 1953).

Calculations of rotational-noise and vortex-noise levels at a distance of 300 ft. for engine ratings of 1,000 to 10,000 h.p. have been made for a large number of propellers in static operation. Propellers with three, four, six, and eight blades and diameters of 8, 12, 16, and 20 ft. are considered. Applications of the data to tandem and dual-rotating configurations are given and the supersonic-type propeller is also briefly considered.—(29.8).

RESEARCH

Notes on the technique employed at the R.A.E. in low-speed wind-tunnel tests in the period 1939-1945. *F. B. Bradfield. R. & M.* 2556 (*October* 1947, *published* 1952).

Very little was recorded during the war years on the details of technique used in low-speed wind-tunnel tests. The size and type of tunnel used during this period will remain in use at firms and colleges for some time after newer equipment is available at research establishments, so it has been decided to issue some record of the technique in use at the R.A.E. during the war years, both with a view to establishing a standard technique where it is satisfactory, and to consider weaknesses where it has failed.—(31.2.1 × 1.12).

STRUCTURES

Loads

A revised formula for the calculation of gust loads. K. G. Pratt. N.A.C.A. Technical Note 2964 (*June* 1953).

A revised gust-load formula with a new gust factor is derived to replace the gust-load formula widely used in gust studies. The revised formula utilises the same principles and retains the same simple form of the original formula. The gust factor is calculated on the basis of a one-minus-cosine gust shape and is presented as a function of mass ratio as compared to the ramp gust shape and wing loading, respectively, used for the alleviation factor.—(33.1 × 1.6).

Theory and Analysis

General instability of simply supported rectangular plates. J. R. M. Radok. A.R.L. Australia. Report SM. 204 (*March* 1953).

The problem of lateral bending of thin plates, stiffened by line reinforcements, is reduced to the limiting case of uniform plates under triangular loading, distributed over a strip, when the width of the strip tends to zero. Using the basic solutions, arising from this limiting process, the characteristic equations for the compressive buckling loads of rectangular plates, reinforced by stringers, by ribs, by distributed stringers and discrete ribs are deduced.—(33.2.1).

A note on the buckling of simply supported parallelogram plates. J. Guest and J. P. O. Silberstein. A.R.L. Australia. Structures and Materials Note 204 (*May* 1953).

The buckling load of simply supported parallelogram plates is re-examined and the results previously obtained by Anderson and Guest are considerably improved for the case of a rhombus with a skewness of 30°. A brief discussion is included of the suitability of various sets of functions for use with energy methods.—(33.2.1).

The local instability of compression members built up from flat plates. A. van der Neut. Delft V.T.H. Report 47 (*August* 1952).

The problem of local instability of structures, composed of flat plates, rigidly connected along the longitudinal edges, is essentially to find the interaction between the composing plates, resulting in equality of buckling stresses and wave lengths for all individual plates. With complicated structures like plates reinforced by longitudinal stiffeners, the computations required for determining the exact solution are highly laborious. This paper presents an exact method, which reduces the amount of numerical work by applying nomograms, giving the relation between buckling stress, wave length and edge restraint. The method can be applied to structures composed of an arbitrary number of walls.—(33.2.1).

Effect of variation in rivet strength on the average stress at maximum load for aluminum-alloy, flat, Z-stiffened compression panels that fail by local buckling. N. F. Dow, W. A. Hickman and B. W. Rosen. N.A.C.A. Technical Note 2963 (*June* 1953).

A study is made of the effect of variation in rivet strength on the average stress at maximum load for 75S-T6 aluminium-alloy, flat, Z-stiffened compression panels that fail by local buckling. A curve is presented for the determination of the relationship between strength, diameter, and pitch of the rivets and the strength of stiffened, flat compression panels of such proportions that failure is by local buckling.—(33.2.3).

APPOINTMENTS

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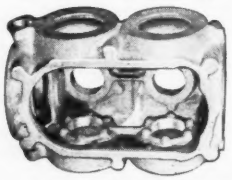

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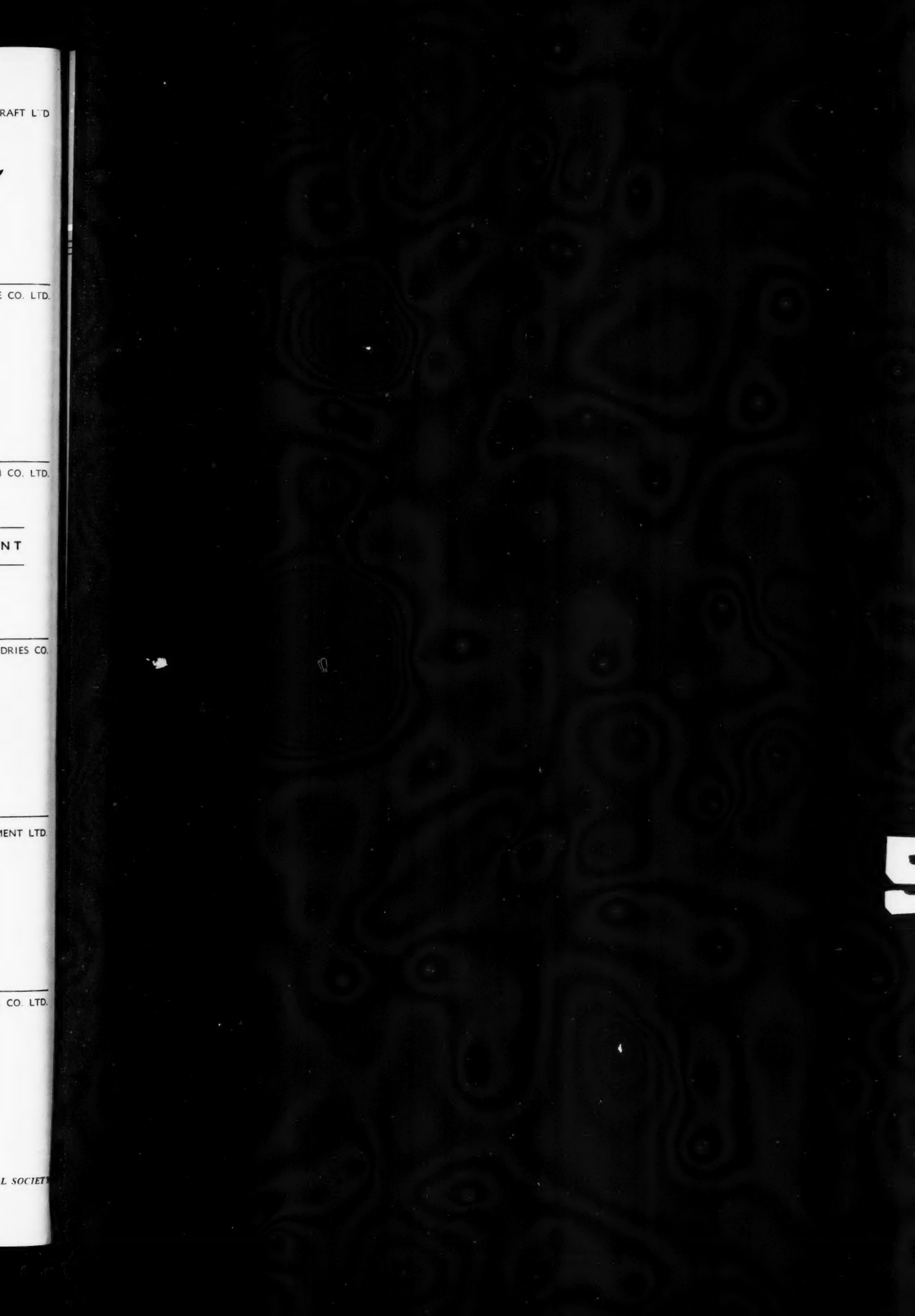
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